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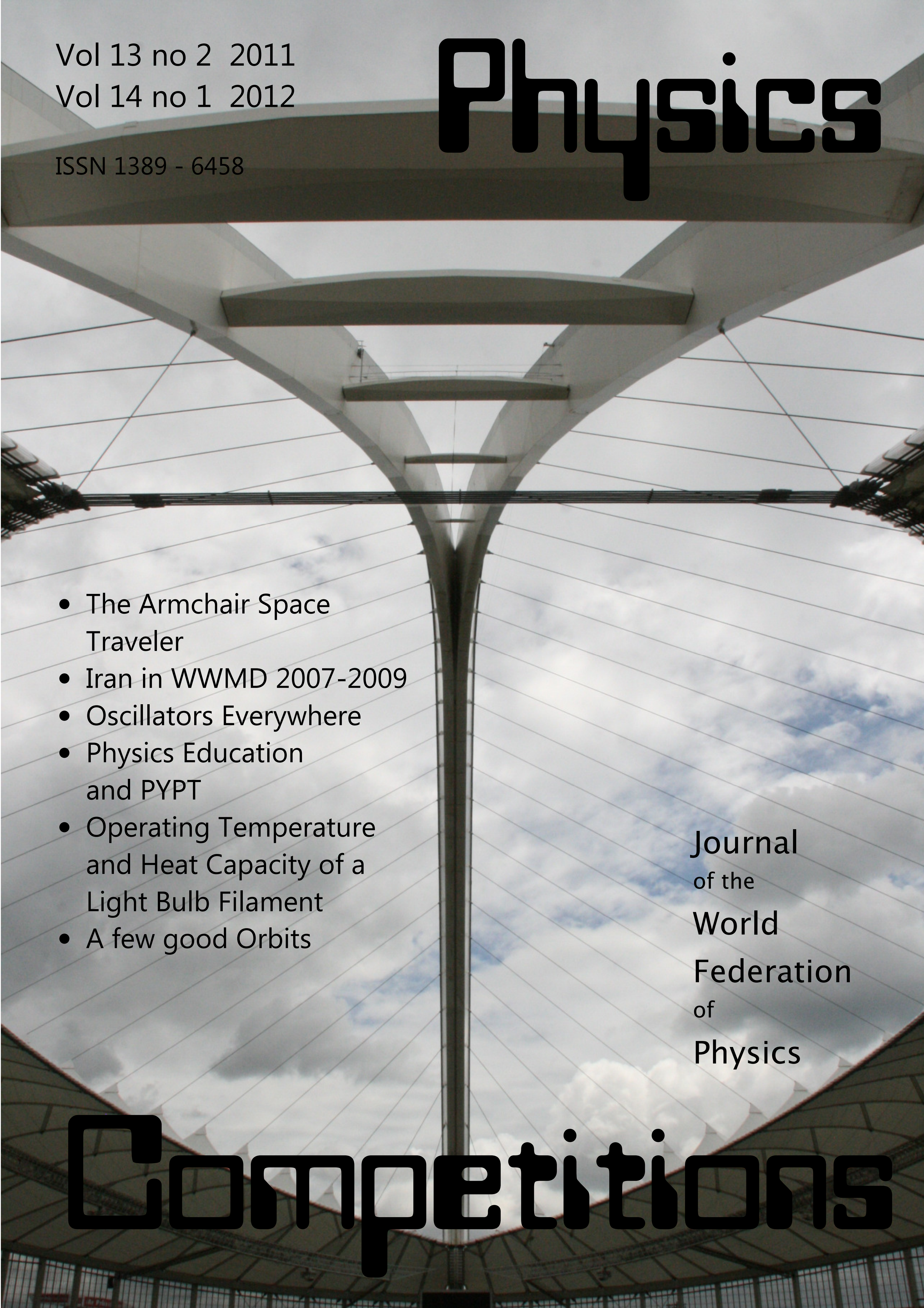
Physics



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Journal of the World Federation of Physics

Competitions

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Physics Competitions

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Department of Physics and Astronomy
6224 Agricultural Road
Vancouver B.C.
Canada V6T 1Z1
kotlicki@physics.ubc.ca

The aims of the Federation are:

1. To promote excellence in, and research associated with, physics education through the use of school physics competitions;

2. To promote meetings and conferences where persons interested in physics contests can exchange and develop ideas of use in their countries;

3. To provide opportunities for the exchanging of information for physics education through published material, notably through the Journal of the Federation;

4. To recognize through the WFPhC Awards system persons who have made notable contributions to physics education through physical challenge around the world;

5. To organize assistance provided by countries with developed system for competitions for countries attempting to develop competitions;

6. To promote physics and to encourage young physicists.

EDITORIAL

In April 2012 the 5th Congress took place in a holiday resort near Nijmegen in the Netherlands. We had chosen to have this Congress just before the International Conference for Young Scientists (ICYS) so that the participants would have the chance to attend both, the Congress and the Conference.
The theme of the Congress was "Interdisciplinarity" and we will report on it in the next Journal.

Following our statutes, there were elections during the Congress. The result was a new situation:
Hans Jordens from the Netherlands, who had acted as WFPhC-president from the beginning of the Federation in the year 2002, had decided to retire from his position and wanted to be "just" an "ordinary member" of the Federation in future.

Dear Hans Jordens,
We have to say "Thank you very much !". You played an important role in the Federation. It was you who founded it, together with Waldemar Gorzkowski, and after Waldemar passed away very unexpectedly in 2007, you followed Waldemar in his job as IPHO-president in 2008. In all those years you did a great job in our Federation !

At the same time Zsuzsanna Rajkovits from Hungary, up to this very date acting as vice-president, wanted to stop her work in the Federation.

Dear Zsuzsanna,
You did an important job, very often more in the background, and we have to say "Thank you very much!" for all your activities.
Zsuzsanna Rajkovits was chosen from the Awards Committee to get the prize of the year 2010. It was given to her during the 4-th Congress, and later in this journal you can find an article about that event.

Matti Rajamäki from Finland, being the head of the Award Committee, also wanted to terminate his activities.

Dear Matti,
Once again we have to say "Thank you so much!" for your activities, both in this Committee and in the Executive Committee.

Yohanes Surya from Indonesia also wanted to stop his activities in the Executive Committee. That´s why we have to say, once again: "Dear Yohannes! Thank you so much for all the work you did for our Federation!"

Jan Mostowski from Poland chose not to continue his activities in the Executive Committee and changed over to the Awards Committee.

Dear Jan! Thank you very much for all your commitment !

The Awards Committee decided to give the Award-2012 to Maija Ahtee from Finland. She was head of the Finnish team from the very beginning of Finland´s

participation in the IPHO up to 1999. From this very year up to 2007 she acted as secretary of the International Physics Olympiad. As the former IPHO-president, Waldemar Gorzkowski, passed away during the IPHO-2007, Maija Ahtee was acting president up to 2008, when the new president, Hans Jordens, was elected. From that date up to 2009 Maija Ahtee worked as secretary again. Then she stopped her engagements in the IPHO.
Maija Ahtee took an important role in the IPHO, very often more in the background than on stage. It is planned to hand the prize over to Maija during the IPHO-2012 in Estonia. She will attend this IPHO as a special guest.

Hence there is a new situation in our Federation with new people in several functions, which one can read at the beginning of this journal.
I am sure that the new crew will continue all the activities of the Federation in the sense of the founders. I very well remember all the discussions we had in 2002 in Bali, when the Federation was founded and will do my best to act in this sense.

Helmuth Mayr
June 2012
Helmuth.mayr@chello.at

The WFPhC- award 2010 Dr. Zsuzsanna Rajkovits

During the congress of the Federation it was a great pleasure for me to hand the WFPhC-award 2010 to Dr. Zsuzsanna Rajkovits from the Öetvös Lorand University of Budapest, Hungary, for her outstanding achievements in the education of physics. For me the pleasure was double because I think Dr.Rajkovits, who largely contributed and still contributes to the aims of the Federation, not only deserves the prize, but for me it was also because of personal reasons that I was happy about her to receive the award. More than two decades ago we met for the first time when we were visiting the museum of the Kremlin as part of an excursion of the International Young Physicists' Tournament. Soon it became clear that our ideas about education and most of all our mutual fascination for physics, created a natural bond between us both. That bond has remained unchanged ever since. From very close I know how much effort and enthusiasm Zsuzsa puts into her work for physics education. Her contributions in various competitions as well as in presentations and articles are numerous.

In 1994 Zsuzsa created, together with colleague Dr. L. Markovich, a new competition called the International Conference of Young Scientists. This competition aims to give the floor to youngsters to present the results of their research in a similar way as 'real' researchers do during international conferences, as is stated in article 1.1.: "The International Conference of Young Scientists (ICYS) is essentially an individual competition on scientific research and presentations carried out by school students which are evaluated by the International Jury." Very often the participants show their fascination for the subject they have researched by presentations that can easily meet the standard of professionals. The way in which they discuss about different topics of their presentation not seldom show profound knowledge and enthusiasm. An important aspect of the conference is that the participants are stimulated to meet and make contacts, and in some cases friendships are created for the rest of their lives. The ICYS is a very successful and truly international competition in which now some 200 participants from all over the world participate.

When Zsuzsa was asked to spend her efforts for the World Federation of Physics Competitions, she did not hesitate for a second and agreed to become vice-president, a position which she occupies till today. As a member of the Executive Committee she co-organized the past congresses and ever since than she gave presentations herself during these congresses. Many articles for the journal of the Federation: "Physics Competitions" appeared from her hand.

The decision by the award committee of the WFPhC on who should receive the award in 2010 was unanimous. This decision was welcomed with a warm applause by all the participants of the 4^{th} Congress of the Federation in 2010 in Baske Ostarija, Croatia.

Hans Jordens

The Armchair Space Traveler

Jim Boyd[1], Don Hinson[2] & Richard J. Palmaccio[3]

[1]Mathematics Department, St. Christopher's School, Virginia, USA
[2]74 Asheridge Rd, CHESHAM HP5 2PY, England, UK
[3]Mathematics Department, St. Sebastian's School, Needham, MA USA

Abstract
This is an article presents a vicarious adventure into space through the examination of various problems in physics. The first takes the armchair traveler to the moon. Upon arrival there, the second problem concerns a study of elements in a sample taken from the moon's surface. The next move is a journey to Jupiter as the third problem studies the "slingshot" effect whereby the gravity of Jupiter is used to accelerate a spaceship past that planet. Finally, our armchair journey takes a cosmic leap with the final problem dealing with the use of a fusion reaction to gain more acceleration for a trip to the stars.

1. Introduction

The Age of Space Travel was inaugurated in 1957 when Russian engineers placed their first Sputnik in orbit around the earth. Shortly thereafter, they sent a dog around the world aboard their second Sputnik. Thus the first space passenger to orbit the Earth was canine rather than human. Then, in 1961, the Russian cosmonaut Yuri Gagarin was launched into a single orbit, thus qualifying as the first human space traveler (of whom we are aware). The first space venture other than "around the World and back again" occurred in 1965 when the unmanned Russian craft Venera 3 crashed onto the surface of Venus. This flight was followed in 1966 by the soft landing of the American Surveyor 1 on the Moon.

True space travel in which a human being visited a body in space other than the Earth began in 1969 with the flight of the American Apollo 11 which carried Neil Armstrong, Buzz Aldrin, and Michael Collins to the Moon. While Collins remained behind circling the Moon in the command module, the lunar landing craft carried Armstrong and Aldrin to the lunar surface where they safely disembarked to take their triumphal walks. The world watched the space adventure and the safe return of the three astronauts on its television screens.

Just as the expeditions of the Nineteenth Century and early Twentieth Century explorers who traveled to the unknown regions of planet Earth stimulated interest in both real and vicarious terrestrial travel, so the voyages of the first cosmonauts and astronauts have inspired actual and armchair space travel. Armchair space travel is the safer and more comfortable of the two varieties, but with paper, pencil, and calculator, we can investigate and solve problems that might confront real space travelers while remaining in the comfort of our own studies. Thus armchair space travel can possess a reality beyond the reading of descriptions of journeys to the source of the Nile, or treks across Antarctica to the South Pole.

2. The Problems

The first problem has three parts.

<u>Problem 1</u>. (Flying to the Moon)

A. What is the escape velocity for a rocket launched from the surface of the Earth?

B. What is the minimum energy per kilogram required to deliver a spacecraft to the Moon from the surface of the Earth?

C. How much kinetic energy per kilogram must be dissipated (by retro-rockets) in order that the spacecraft make a soft landing on the lunar surface?

Before proceeding with our solutions, we list the numerical values in S.I. units of the physical constants that will appear in our calculations:

g = gravitational acceleration at the Earth's surface = 9.81
G = universal gravitational constant = 6.672×10^{-11}
$R(0)$ = distance between the centers of the Earth and Moon = 3.844×10^{8}
$R(E)$ = radius of the Earth = 6.37×10^{6}
$R(M)$ = radius of the Moon = 1.738×10^{6}
M = mass of the Earth = 5.974×10^{24}
m = mass of the Moon = 0.735×10^{24}

<u>Solution</u>.

A. Let us assume that all of the rocket's kinetic energy T is delivered to it at a single shot at the instant of lift-off and that T is sufficient to carry the rocket beyond the Earth's gravitational field (i.e. to "infinity"). Furthermore, let us assume that air resistance does not act on the rocket as it travels through the Earth's atmosphere. Then

$$T = \left(\frac{1}{2}\right)\mu v^2 \geq \frac{GM\mu}{R(E)} - 0 = \frac{GM\mu}{R(E)}$$

where μ and v represent the mass of the rocket after its fuel has been spent, and the initial velocity of the rocket, respectively.

If v is taken to be the escape velocity, then

$$\left(\frac{1}{2}\right)\mu v^2 = \frac{GM\mu}{R(E)} \tag{1}$$

Equation (1) states that all of the kinetic energy delivered to the rocket at lift-off is converted to gravitational potential energy. Since $g = \frac{GM}{R(E)^2}$, equation (1) may be rewritten as $v^2 = 2gR(E)$. Solving this equation for v and using the values given above for the relevant constants, we find the escape velocity to be $v \approx 11.19\ {}^{km}/_{s}$.

B. Since the spacecraft does not have to leave the earth's gravitational field, but only travel to the Moon, and since the Moon's gravitational pull on the spacecraft will be helpful in the flight, the kinetic energy required to send the spacecraft from the surface of the Earth to the surface of the Moon is less than that associated with the escape velocity.
To simplify our thinking, les us imagine that the spacecraft travels along the line of centers of the Earth and the Moon. We then note that there is a point, say P, on this line at which the gravitational attractions by the Earth and the Moon precisely cancel one another out. Now let the distance of P from the center of the Earth be denoted by r. Since the gravitational forces by the Earth and the Moon on the spacecraft are of equal size but in opposite directions at P, we may write that $\frac{GM}{r^2} = \frac{Gm}{(R(0)-r)^2}$ which implies that $\frac{R(0)}{r} = 1 + \sqrt{m/M} \approx 1.111$. Thus $r \approx 3.46 \times 10^8$ m.

The work performed on 1 kilogram of mass in carrying it from the surface of the Earth to point P is given by

$$T' = \int_{R(E)}^{r}\left(\frac{GM}{x^2} - \frac{GM}{(R(0)-x)^2}\right)dx = \left(\frac{-GM}{r} + \frac{GM}{R(E)}\right) - \left(\frac{GM}{R(0)-r} - \frac{GM}{R(0)-R(E)}\right) \tag{2}$$

Evaluating the right hand side of equation (2) by substituting the value of r as found above and the values of the physical constants listed at the beginning of our solution yields the kinetic energy converted to gravitational potential energy as 1 kilogram is lifted from the surface of the Earth to point P. Any kinetic energy in excess of T' will carry the kilogram to P and, thereafter, the kilogram will plummet to the lunar surface under the gravitational attraction of the Moon. The value of T' may be taken to be the minimum value of initial kinetic energy per kilogram which will send the spacecraft to the Moon. We find that $T' = 6.146 \times 10^7$ J/kg.
The initial velocity associated with this kinetic energy is given by $v' = \sqrt{2T'} \approx 11.09$ km/s which is, as argued above, slightly less than the escape velocity calculated in part A of our solution.

C. The distance of the "zero gravity" point P from the center of the Moon is given by $r' = R(0) - r \approx 3.84 \times 10^7$ m. The work required to lift 1 kilogram from the surface of the Moon to P is

$$T'' = \int_{R(M)}^{r}\left(\frac{GM}{x^2} - \frac{GM}{(R(0)-x)^2}\right)dx = \left(\frac{-GM}{r} + \frac{GM}{R(M)}\right) - \left(\frac{GM}{R(0)-r} - \frac{GM}{R(0)-R(M)}\right) \tag{3}$$

Evaluating T'' as we evaluated T', we find that $T'' = 2.584 \times 10^7$ J/kg. This is also the kinetic energy per kilogram that should be dissipated if the spacecraft is to touch down on the Moon's surface with zero velocity. We note in conclusion that the minimum lift-off velocity that will send the kilogram from the Moon's surface back to P is $\sqrt{2T''} \approx 2.273$ km/s.

Once we reach the Moon, we ought to do some science. One reason for going to the Moon might be to examine the materials on its surface. So, our next problem which comes in two parts suggests a project that we might pursue.

Getty Images

Problem 2. (Doing Science on the Moon).

A. A sample taken from the surface contained 4.0 grams of potassium of which 0.01% was in the form of the isotope $^{40}_{19}K$ which decays into argon with a half-life of 1.2 x 10^9 years. When the sample was heated in a vacuum, 2.4 cubic centimeters of argon at a temperature of 20° C and a pressure of 101.3 kPa were released. Calculate from these data a minimum age for the Moon. Assume that all of the argon in the sample was produced by the decay of potassium.

B. Alpha particles were used to bombard a small region of the lunar surface near the site from which the sample of part A above was taken. The purpose of the bombardment was to identify the chemical elements in the Moon's soil. In one experiment, the maximum kinetic energy of the recoiling alpha particles was 37% of their incident kinetic energy. Identify the element with whose nuclei the alpha particles collided.

Solution.

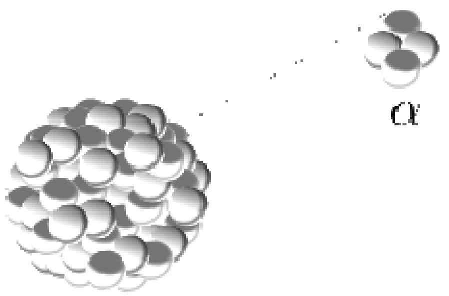
A. The 4 grams of potassium constitutes 0.1 mole which implies that there is 10^{-5} of a mole of ${}^{40}_{19}K$. Sample calculations based on the Ideal Gas Law, $Pv = nRT$, yield the number of moles of argon released:

$$(101.3 \times 10^3) \times (2.4 \times 10^{-6}) = n(8.31 \times 293) \rightarrow n = 10^{-4} mole$$

We can now use the equation of exponential decay

$$A(t) = A(0)e^{-\lambda t} \tag{4}$$

to compute the age of the Moon based on the assumptions that we have made. In this equation, $A(0)$ represents the amount of ${}^{40}_{19}K$ in the sample at $t = 0$ (the "instant" at which the Moon was formed). In our problem, $A(0) = 10^{-5} + 10^{-4} = 11 \times 10^{-5}$ mole. The parameter λ can be calculated from the half-life 1.2×10^9 years or radioactive potassium by noting that $\lambda = (\ln 2)/(1.2 \times 10^9) = 5.776 \times 10^{-10}$. Making the appropriate substitutions into equation (4) and letting $A(t) = 10^{-5}$, we solve the equation to find that $t = 4.15 \times 10^9$ years. This value is the estimated age of the Moon.



B. The maximum recoil energy will occur when an alpha particle suffers a head-on collision with the nucleus of an atom under bombardment. In this case, the approach and separation of the alpha particle and nucleus before and after collision will take place on a fixed line. Furthermore, we shall assume that, before collision, the nucleus was at rest.

We take the mass of the alpha particle to be 4 mass units and the mass of the nucleus that we wish to identify to be M mass units. We denote the velocity of the alpha particle before it collides with the stationary nucleus by v and the velocity of the nucleus after the collision by u. The Law of Conservation of Energy implies that

$$\left(\frac{1}{2}\right)4v^2 = (0.37)\left(\frac{1}{2}\right)4v^2 + \left(\frac{1}{2}\right)Mu^2.$$

Simplification yields

$$M = 2.52\left(\frac{v}{u}\right)^2. \tag{5}$$

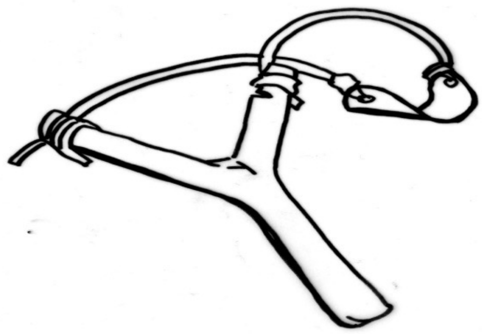
Since the kinetic energy of the alpha particle after the collision is 37% of the incident kinetic energy, its recoil velocity must be $\sqrt{0.37}\ v = 0.6087\ v$. Then the Law of Conservation of Momentum implies that

$$4v = Mu - 4(0.608)v\ .$$

This result in turn implies that

$$M = 6.432\left(\frac{v}{u}\right). \tag{6}$$

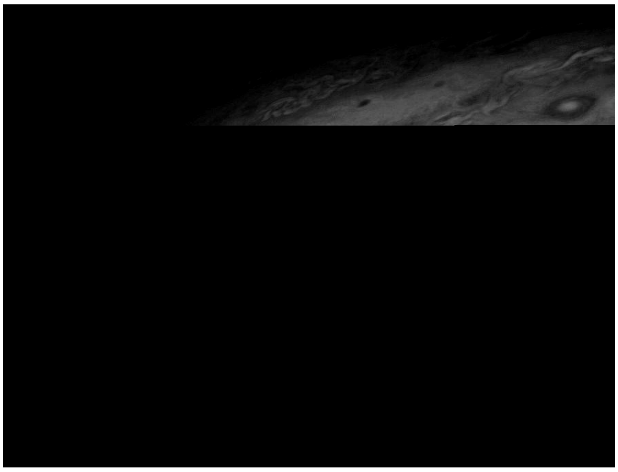
Solving equations (5) and (6) simultaneously yields $M = 16.4$ mass units. Thus we may infer that the element with which the alpha particle collided was oxygen.



Maximizing the efficient use of fuel in space is of prime importance. Robinson Crusoe's isolation would be as nothing compared to being lost in space. All ways of saving fuel ought to be investigated and exploited if possible. One concept is that of the "slingshot" principle. The idea is that when a spacecraft is traveling in the vicinity of a planet which is itself approaching the spacecraft (with respect to the rest frame of the fixed stars), a transfer of energy can occur which results in an increase in the speed of the spacecraft. Of course, the pull of the planet's gravitational field will slow the spacecraft down somewhat, but the velocity with which the spacecraft departs may still exceed the velocity with which it approached the planet.

A rough analogy may make the "slingshot" effect understandable to our armchair traveler. Let us imagine the level swing of a massive bat as it moves to strike an incoming ball. We take the ratio of the masses of the bat and ball to be comparable to the ratio of the masses of the spacecraft and planet. That is, the mass of the bat may be taken to be infinitely great. Let us assume that the collision between the bat and ball is perfectly elastic. Then, in the rest frame of the bat, the ball will rebound without loss of kinetic energy. The velocity v of the ball will be reversed in direction by its collision with the bat, but the magnitude of the ball's velocity will remain constant. Returning to the rest frame fixed on the baseball diamond, we see an increase in the magnitude of the velocity of the ball as its speed before the collision is reduced by the speed of the bat while its speed after the collision is increased by the same amount.

Thus the speed of the ball is increased by $2u$ where u is the speed of the bat. The forces developed during the collision were contact forces. In the interaction between the spacecraft and the planet, the forces are gravitational and contact does not occur since a crash landing would be undesirable. However, the basic idea is the same for the ball-bat and spacecraft-planet interactions.



Problem 3. (Getting a Lift).

A spaceship with a speed of v = 15 km/s approaches the planet Jupiter which is moving toward it at a speed of u = 13 km/s with respect to the fixed stars. The approach is oblique and at a distance so great that gravitational attractions may be neglected. The path of the spacecraft makes an angle of 30 degrees with the direction of the velocity of the oncoming planet. After being attracted and deflected by the planet, the spacecraft emerges from Jupiter's influence with its course altered by 300 degrees as it heads away at the angle of 30 degrees on the other side of Jupiter's direction. Find the gain in speed acquired by the spacecraft during its interaction with the planet.

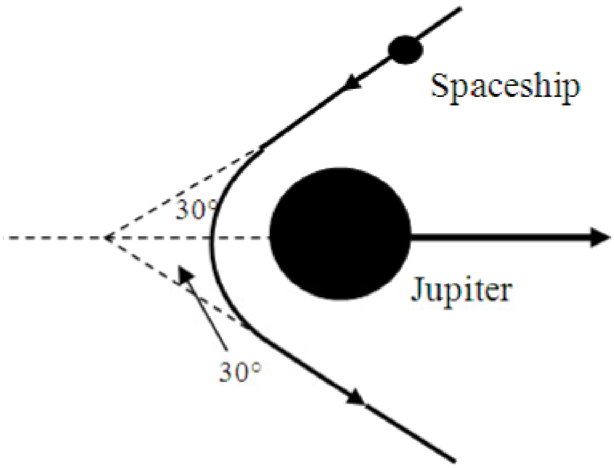


Figure 1. Jupiter's "Slingshot".

The geometry of the problem is suggested by Figure 1.

Solution.

Resolve the initial velocity of the spaceship into components parallel and perpendicular to the planet's path. Then

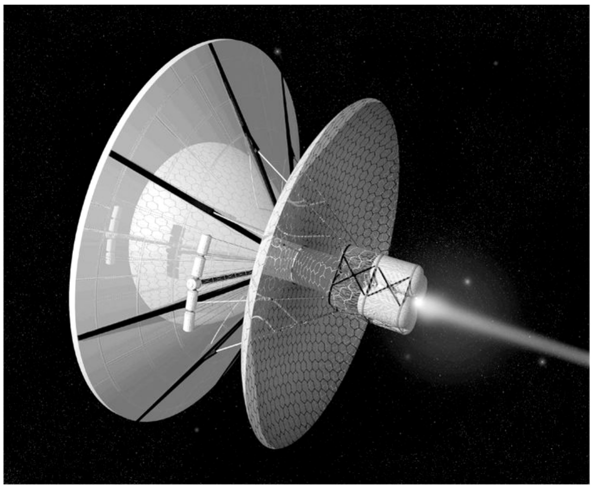
v(perpendicular, 0) = 15 sin 30° = 7.5 and v(parallel, 0) = 15 cos 30° = 12.99 .

The component of velocity perpendicular to the planet's path will be unaffected by the interaction between the spaceship and the planet. By the argument advanced in our analogy of the batted ball, the direction of the parallel component of velocity will be reversed and its magnitude increased by 2(13) = 26. The components of velocity as the spaceship heads away from Jupiter will be

v(perpendicular, 1) = 15 sin 30° = 7.5 and

v(parallel, 1) = -15 cos 30° – 26 = -38.99 .

The gain in speed will be $\sqrt{7.5^2 + 38.99^2} - \sqrt{7.5^2 + 12.99^2} = 24.71\ km/s$. Thus Jupiter's "slingshot" increases the spaceship's speed by almost 25 km/s without the expenditure of any fuel at all!



The armchair imagination is completely free. So here is a really far out idea for saving fuel. Might not an interstellar spaceship collect protons in space as fuel? The protons would be fed into a fusion reactor to generate kinetic energy to accelerate the ship. Those interested in collecting protons should know that their density in space is n = 0.1 proton per cubic centimeter which is 10^5 protons per cubic meter.

Problem 4. (Accelerating the Spaceship).

How large a scoop would have to be designed to collect enough protons to develop an acceleration a = 10 m/s^2 for a 3500 tonne ship if f = 0.01 of the energy of the fusion reactor is converted to kinetic energy of the spaceship? Assume that 100% of the kinetic energy produced is converted to thrust. For protons to be captured they must incident on the surface of the scoop mechanism.
We need to recall that a mass of 1 tonne is M = 1000g, the mass of a proton is m = 1.7 x 10^{-27} kg, and the speed of light is c = 3 x 10^8 m/s in order to complete our calculations.

Solution.

Let us suppose that the velocity of the spaceship is v and that trailing behind the spaceship is the scoop which has a cross-sectional surface area A presented perpendicular to the direction of the spaceship's travel. The volume of space swept for protons per second by the scoop will be Av and the mass of protons collected per second will be $nmAv$ = 1.7 x 10^{-22} Av kg/s. Using Einstein's relationship for the energy equivalence of mass, we find that the energy collected is $1.7 \times 10^{-22} Avc^2 = 1.53 \times 10^{-5} Av$ J/s. This result represents an input of power. Of this input, $f(1.53 \times 10^{-5} Av) = 1.53 \times 10^{-7} Av$ J/s is converted to thrust.

The power required to develop the thrust to give the ship an acceleration of $a = 10$ m/s^2 is $3500Mav = 3.5 \times 10^7 v$ J/s. Equating the two values of power just obtained, we find that

$$1.53 \times 10^{-7} Av = 3.5 \times 10^7 v$$

implying that $A = 2.29 \times 10^{14}$ square meters, an enormous surface area indeed. If the scoop were circular, its radius would be $\sqrt{(2.29 \times 10^{14})/\pi} = 8.538 \times 10^6$ meters = 8537 km. Although the idea was an interesting one, such a scoop would be completely impractical.

Our armchair astronaut was a bit disappointed that his idea turned out to be unrealistic. However, he was not one to give up so easily. He had read about "wormholes" that might connect distant parts of the universe. Perhaps he could find an entrance. And then, in just the briefest of moments before it closed, he could ⋯

Iran in WWMD 2007-2009

Dina Izadi

Ariaian Young Innovative Minds Institute (AYIMI), Tehran, Iran
http://www.ayimi.org, *info@ayimi.org*

1. Introduction

World Water Monitoring Day (WWMD) is an international education and outreach program by coordination of Water Monitoring Day, the Water Environment Federation (WEF) and the International Water Association (IWA) that builds public awareness and involvement in protecting water resources around the world by engaging citizens to conduct basic monitoring of their local water bodies. World Water Monitoring Day activities takes place from March through December each year and December 31 served as the deadline for reporting. All the participants sampled their local lakes, streams, rivers, ponds, estuaries and other Water bodies for four key water quality indicators: dissolved oxygen (DO), pH, temperature and turbidity. Some groups also monitored for the presence of certain macro invertebrates such as dragonflies, mayflies and scuds. Samples were taken in a range of settings—agricultural, commercial, residential and industrial—on six continents. A total of 122,599 participants monitored sites worldwide, which represents a 67% increase over 2008. In Iran we have started this program since 2007. Each year we send some educational kits to the universities and schools that are willing to participate in these practical activities. These kits are sent by the WWMD organizers. We organized the 1st scientific workshop in 2009 and one of the topics was WWMD. Participants learned how to use the kits and how to do in situ experiments.

2. Monitoring Four Rivers in Iran for WWMD™, 2007

Sikhoran, Roudan, Karaj and Jagerood Rivers.

In this project students from Bandar Abbas Azad University (Flora Mohamadizade,Marzie Haj Keramadini, Abbas Barkhordari, Shadi Khatami, Maria Mohamadizade, Parviz Tavakoli) and 10 students (Atena Shirdastian, Shiva Mahdaviseresht, Noshad Khosravi, Golnoosh Azarbakhsh, Yasaman Mehrabi, Orkide Olang, Mahsa Kave, Negin Amini, Farimah Ramezanpoor, Niloofar Keramati) from Aboureihan educational complex took part.

Hormozgan province covering an area of 68,476 km2 is located southern Iran, north of the Persian Gulf. The main part of the province is covered by mountainous regions. The Zagross mountain range extends from northeast to southeast and terminates to lime and sandy hills, highlands, as well as coastal lowlands, parallel to the Persian Gulf and Oman Sea. Roudan and Sikhoran are ancient geographical regions of this province. The measurements in Karaj River were carried out in a part of this river in Gachsar to Chaloos road (about 6 kilometers after Karaj) on a sunny day and Jagerood measurements near Latia Dam (Fig. 1).

Figure 1. In situ experiments in Iran, 2007

3. Results in 2007

The following figures represent the results for 2007 based upon the four WWMD™ water quality parameters in Iran. These are listed by country and do not constitute a completely thorough and accurate portrayal of the health of the world's water. Credible water quality sampling requires using standard quality assurance protocols and is conducted with trained volunteer monitoring groups and professionals around the world (Table 1).

Table 1. Results in Iran , 2007

Country	Total Sites	Total Participants	Average DO (PPM)	Average Temperature (C)	Average pH	Average Turbidity (JTU)
Iran	6	46	4.00	15.72	6.78	13.33

4. Numerical Summary 2007

Forty-three countries participated in World Water Monitoring Day™ 2007 and 46,117 participants reported data from a total of 3,544 sites. Test Kits Distributed were about 10,636 and monitored sites were 3,544 and Total Participants were 46,117. The following figures represent the results of WWMD™ 2007 for the continents of North America, South America, Europe, Asia, Africa and Australia based upon the four WWMD™ water quality parameters (table 2).

Table 2. Water quality parameters 2007 (http://www.worldwatermonitoringday.org/2007)

Continent	DO	pH	Temperature	Turbidity
Africa	4.75	7.88	23.35	28.78
Asia	4.67	7.15	23.04	39.83
Australia	4.41	7.00	17.50	21.82
Europe	5.07	7.35	10.44	6.07
North America	5.31	7.68	18.99	7.44
South America	6.19	7.53	20.01	22.71

The in situ experiments have been done in different countries to get more details about water quality in the world (Fig. 2).

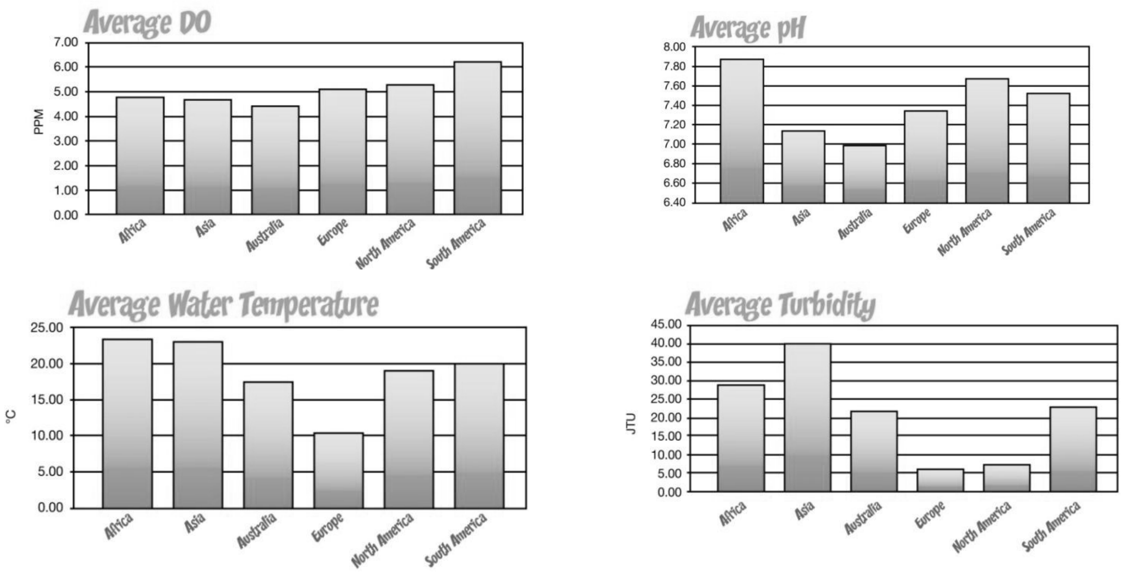


Fig. 2 Average DO, PH, Temperature and Turbidity, 2007(http://www.worldwatermonitoringday.org/2007)

5. Monitoring different Rivers in Iran for WWMD™ ,2009

In 2009, over 120,000 people in 81 countries monitored their local waterways. The 1st workshop on Nov. 20, 2009 was organized by the Ariaian Young Innovative Minds Institute (AYIMI) with cooperation of Iranian Society of Marine Science and Technology (ISMST) and the Scientific Societies of Amirkabir University of Technology (AUT) and students registered to receive the kits.

Students from Rahe Roshd education complex in Tehran (Jeiran Joorabchi, Tina Aghajooni, Sadaf Farhoodi, Diba Bagheri, Golsa Saadati, Parinaz Sadoughi, Negar Shariat, Ava Mohammadi, Ariyaneh Abedinnejad, Mahrouya Nikoufar, Kiana Abbasi, Sogand Shariatmadari, Mahsa Abbaspour, DonyaMazloumi, Saba Sendani and teamleader Tayari) as a team work and a student from Amin School in Tabriz (Farid Mehri) sent their reports to the AYIMI and we provided all details for WWMD. Their measurements were from the samples of Darake Local River, Karaj lake dam and Darband Local River. Darband and Darake are some kind of recreational centers to visit and Karaj's dam is one of the most important water reservoirs of Tehran. They started sampling on 14th and 15th of December. At both of the rivers were no kinds of extra pollutions in the samples of water. Karaj's dam is a restricted area so the water was clean and with no kind of serious pollutions.

Table 3. Results in Iran, 2009

Country	Participants	Sites	DO (PPM)	pH	Water Temperature (C)	Turbidity (JTU)
Iran	56	23	4.56	7.31	5.79	17.25

By sampling these local waters they found that the water resources and the ingredients are one of the most important elements in their daily lives and should make effort protecting them from any kind of damages and pollutions and they have to know that living without healthy water is impossible.

6. Watershed domain Karaj

Small water area in Karaj of approximately 1115 square kilometers in north-eastern city of Karaj was sampled. The analysis done on that project within Hydrometric Station melting snow 80 percent, which is variable due to having field, 41 percent to 6 percent of the maximum domain of Karaj during various boards of at least mountainous conditions and type of winter precipitation that is often the snow. River Darakeh: These areas in the north-west of Tehran and the median area of Khoshkeh and Velenjak. Domain area is of 2500 hectares of mountain boards and the Altitude 3876 meters above sea level. The first kilometer of the River Valley, which is Darakeh approximate width of 10 m and depth of this part of the average is 4 meters.

Table 4. Water quality parameters in Darband, Darake and Karaj, 2009

Location	Date	Time	Air Temp (°C)	Water Temp (°C)	Tur (JTU)	Do (ppm)	pH	Saturation
Darband creek	2009/12/14	8:26 A.M	0	2	0	8	7	58%
Darake creek	2009/12/14	10:58 A.M	2	2	0	8	8	58%
Karaj dam lake	2009/12/14	4:30 P.M	4	6	40	4	7.5	31%
Darband	2009/12/15	8:13 A.M	2	3	0	8	7	61%
Darake	2009/12/15	10:30 P.M	2	4	0	8	7.5	61%
Karaj	2009/12/15	15:12 P.M	4	6	40	8	7.5	64%

7. Kandowan Area for sampling

Kandowan is a popular town in Eastern Azerbaijan of Iran. There is a river with name of Kandowan Chaei. Also there is a pipe that pumps the water from Kandowan spring that has Mineral Water. Farid Mehri tested both of them individualy. After hiking in Sahand mountain he went to a lake on top of the mountain and tested the water. In Basmenj city there is a river which he tested it too.

Table 5. . Water quality parameters in Azerbaijan, 2009

Location	Date	Coordinates	Air Temp (˚C)	Water Temp (˚C)	Tur. (JTU)	Do (ppm)	pH
Kandowan River	2009/12/25	37.81346.3113	3	2.5	50.000	4	8
Kandowan Spring (Mineral Water)	2009/12/25	37.73146.3345	3	4	0	3	7
The lake on top of the Sahand mountain	2009/12/25	37.726746.3465	-2	2.3	0	2	6
Basmenj river	2009/12/25	37.985946.4921	8	6	30.000	0	6
Jeghateh chaei	2009/12/26	36.971246.1149	14	10.6	30.000	4	8
Zarrine	2009/12/26	36.98246.1091	14	9.3	6.660	8	8

Fig. 3. In situ experiments in Iran, 2009

8. Numerical Summary 2009

Over 120000 people participated in World Water Monitoring Day™ 2009. Test Kits Distributed were about 16000 and monitored sites were 8000. The following figures represent the results of WWMD™ 2009 for the continents of North America, South America, Europe, Asia, Africa and Australia based upon the four WWMD™ water quality parameters (table 6)

Table 6. Water quality parameters, 2009(http://www.worldwatermonitoringday.org/2009)

Continent	Sites in Sample	DO (PPM)	pH	Temperature (°C)	Turbidity (JTU)
Africa	229	4.43	7.36	22.99	29.03
Asia	1,420	3.81	7.38	24.81	27.06
Australia/Oceania	18	5.95	7.00	16.54	9.24
Europe	2,016	4.83	7.21	13.60	17.08
North America	4,305	7.71	7.35	17.43	16.27
South America	102	6.96	7.58	12.27	16.38

The average PH, DO, temperature and turbidity were measured to compare water quality (Fig. 4).

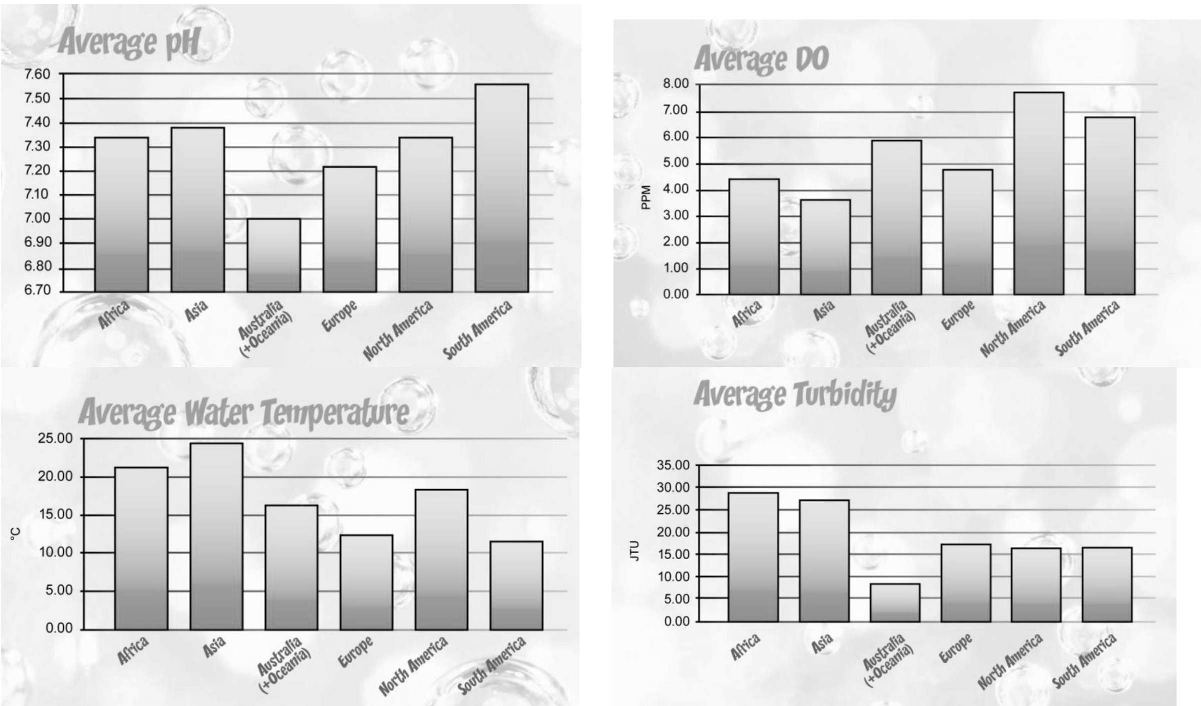


Fig. 4.Average DO, PH, Temperature and Turbidity, 2009 (http://www.worldwatermonitoringday.org/2009)

9. Conclusions

The actual monitoring shows some human activities like washing and swimming along the streams causes pollutions and people need healthy water to drink. Surveys at various sites included measurements for temperature, pH, dissolved oxygen and turbidity are done by monitoring equipments supplied by the World Water Monitoring Day program. The testing water exists in the world help us to know what is accessible to us, and why it is important to conserve and protect our water.

Acknowledgements

It is nice to thank students from Bandar Abbas Azad University, Aboureihan and Rahe Roshd educational complex in Tehran and from Amin School in Azarbaijan which took part in this project in 2007 and 2009.

References: http://www.worldwatermonitoringday.org/2007
http://www.worldwatermonitoringday.org/2009, http://www.ayimi.org

Oscillators Everywhere

Jim Boyd[1], Richard J. Palmaccio[2] & Willie Yong[3]

[1]*Mathematics Department, St. Christopher's School, Virginia, USA*
[2]*Mathematics Department, St. Sebastian's School, Needham, MA USA*
[3] SCT Publishing, Singapore

Abstract
We present a collection of problems about simple harmonic motion. These problems target students who are preparing for the Asian Physics Olympiads, various national Physics Olympiads, and the International Physics Olympiads. We hope that students will find the problems interesting and useful.

1. Introduction

Physicists have given us a model of the world filled with vibration at all scales in the universe from the smallest, grainiest quantum levels to the vast objects of study in astrophysics which demand a unification of general relativity and quantum mechanics.

The domain of classical physics is the "reasonable" world that we experience directly every day. The objects treated by classical physics are neither too small nor too large and their velocities are comfortably less than the speed of light. That domain might be called the "Goldilocks World" because everything therein is "just right." The ubiquity of vibratory motion holds throughout the "Goldilocks World" in which we live, and the first and often most sensible approximation of classical vibratory motion is simple harmonic oscillation, the projection of uniform circular motion onto a diameter. We suspect that more ink has been expended in writing "$F = ma$" and variants thereof than in writing any other equation of classical physics. A strong case could be argued that "$m(d^2x/dt^2) = -k\,x$" accounts for a considerable portion of that ink. The bobbing of a cork in water, the beating of the wings of birds and insects, the swinging of a pendulum, and even the tremors of the ground that we stand upon in the shock and after-shock of earthquakes are examples of vibrations that we have or may have observed in daily life. Since physicists seem to prefer "oscillation" as a word to "vibration", we feel that the title of our article is justified.

But let us stop for just a moment on our way to our first problem in order to comment on the vibratory motion induced by an earthquake. We urge readers to turn to the Internet to find information about the Yasaka Pagoda in Kyoto, Japan. The first version of the structure was built before 600 CE. From time to time during its history, the pagoda caught fire, burned and was rebuilt. The present structure dates from 1440. The design of the Pagoda which the citizens of Kyoto and visitors to that city see today serves as a model for "earthquake proof" construction. The five stories of the Pagoda are attached independently to a tall, central, vertical pillar. The response of the Pagoda to the energy that it absorbs with the shock of an earthquake is that each of the five levels moves freely about the central pillar with a damped oscillation as the energy is transmitted through the pillar to the ground and reabsorbed there.

The central supporting pillar was first incorporated into the design during a reconstruction a little before 940. Thus builders of long ago took advantage of their understanding of vibratory motion.

2. The Problems

Now, on to the problems! We shall present four problems. We shall give solutions to the first three but simply state the fourth, leaving its solution as a challenge and source of pleasure for our readers.

Problem 1. You are sitting in your garden sipping orange juice on a nice summer afternoon. Let us imagine a bee in search of just the right flower. Having spotted a nectar-laden blossom, the bee hovers above before settling down to gather the sweet, raw material for the hive's production of honey. As the bee hovers in midair, it is the motion of the bee's wings that produces the familiar buzz which we hear. Estimate the frequency of the bee's buzz.

Solution. Let the surface area of each of each of the bee's wings be denoted by A. Let the maximum amplitude of the flapping motion of the bee's wings be denoted by z_o. Let us denote the angular frequency of flapping by ω sec^{-1} and finally, let us denote the density of the air by ρ and the mass of the bee by m.

To help our thinking along, we draw a figure to suggest the bee hovering above a flower.

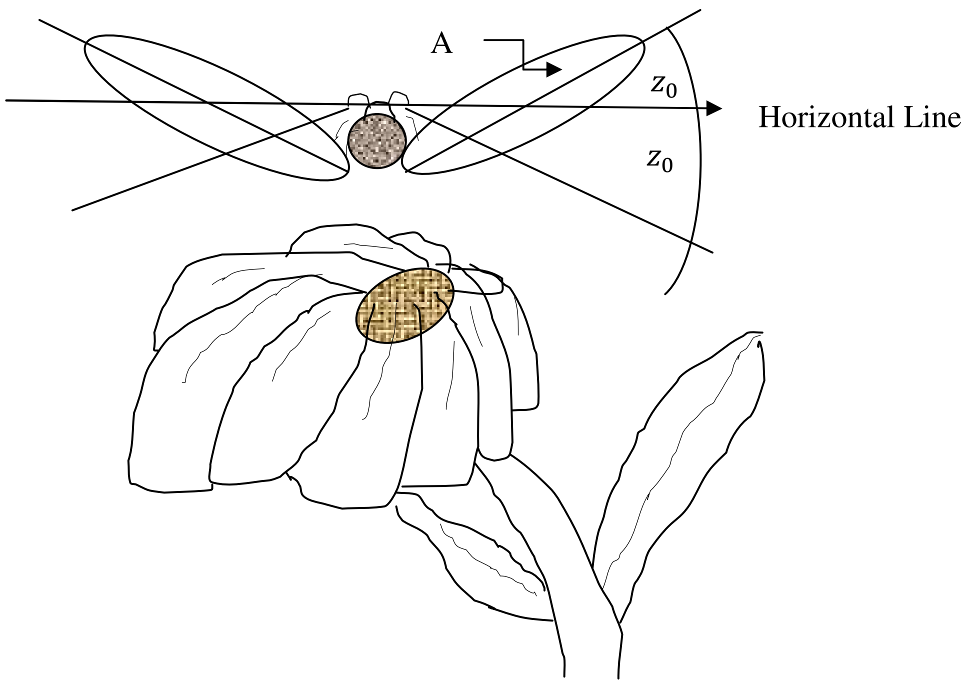


Figure 1. The Bee and the Flower

We must compute the upward force exerted by the air as it supports the weight *mg* of the bee. To do so, we compute the force exerted on the air by the bee during the downward stroke of its wings. The opposite reaction force by the air over the surfaces of the wings supports the weight of the bee.

The volume of air moved by each complete downward stroke of both wings is $2(2z_o)(kA)$ where k is a geometric constant depending on the shape of the region of space swept out by one complete downward stroke of each wing. The first factor of 2 is needed because there are two wings in motion. The second factor of 2 is needed because the tip of each wing travels twice the amplitude with each flap from top to bottom.

Sensible arguments may be advanced that $k \approx \frac{1}{2}$. In any event, we make that assumption and take the volume of air displaced by a complete down stroke of both wings to be $2z_oA$. Then the mass of the displaced air becomes

$$M_{air} = 2\rho A z_0.$$

The assumption that the motion of the wings is simple harmonic is equivalent to assuming that the displacement of each wing may be written as a function of time in the form

$$Z(t) = z_0 \sin \omega t.$$

The acceleration of each wing is then

$$\frac{d^2z}{dt^2} = -z_0\omega^2 \sin \omega t.$$

The magnitude of the force exerted by the bee on the air below it may be approximated by

$$\left| m_{air} \frac{d^2z}{dt^2} \right| = 2\rho A z_0^2 \omega^2 \sin \omega t \qquad (1)$$

with $0 \le \omega t \le \pi$ since we claim that the bee is not affected by the air during the half-period of the upward stroke if its wings.

By Newton's Third Law of Motion, the magnitude of the force that supports the bee during the down stroke will also be given by equation (1). Over the complete period of the motion of the wings, the supporting force is

$$F = \begin{cases} 2\rho A z_0^2 \omega^2 \sin \omega t & \textit{during the down stroke} \\ 0 & \textit{during the up stroke} \end{cases}$$

Next we compute the average value of the supporting force F over a complete period $\frac{2\pi}{\omega}$. We find that.

$$F_{avg} = \frac{\int_0^{\pi/\omega} 2\rho A z_0^2 \omega^2 \sin \omega t \, dt}{(2\pi/\omega)} = \frac{2\rho A z_0^2 \omega^2}{\pi}.$$

We must now make yet another assumption which is that z_o is comparable to the dimensions of the bee's wings. We accomplish that by assuming that $z_o = \sqrt{A}$ so that we may write that

$$F_{avg} = \frac{2\rho A^2 \omega^2}{\pi}.$$

Finally, we equate the average supporting force to the weight of the hovering bee. We obtain

$$\frac{2\rho A^2\omega^2}{\pi} = mg$$

or

$$\omega = \sqrt{\frac{\pi mg}{2\rho A^2}}.$$

We conclude by computing the approximate angular frequency ω for reasonable values of the parameters in our problem. Let m = 0.001 gm, A = 0.170 cm^2, ρ = 0.0013 gm/cm^3, and g = 980 cm/sec^2. We find that $\omega \approx$ 200 sec^{-1} which agrees well with observed buzz frequencies of 200 to 250 sec^{-1}.

Problem 2. Here is a familiar problem. A wooden disc bobs at the surface of an otherwise still pond. The dimensions of the pond are so great with respect to the dimensions of the disc that the level of the water surface remains unchanged as the motion of the disc proceeds. The buoyant force exerted by the water exceeds the weight of the disc when the disc is at its greatest depth and is less than the weight of the disc when it is at the top of its bobbing trajectory. When written, the equation of motion of the disc is easily seen to be that of simple harmonic motion.

Let us now look at a more complicated problem. That problem involves a solid wooden disc of radius a, thickness t, and uniform density ρ_{wood} which floats in a liquid which is contained in a vessel having dimensions comparable to the dimensions of the disc. When the disc is displaced from its equilibrium position, the resulting change in the level of the liquid surface may not be ignored.

Let us say that the density of the liquid is ρ_{liq} and that the vessel containing the liquid is a deep circular cylinder of radius R. We emphasize that we do not assume a << R.

Suppose that the disc is pushed down so that its horizontal upper surface is slightly above the level of the liquid surface and that the disc is then released from rest. Thereafter, the disc bobs up and down without damping by frictional effects while remaining upright at all times.

Determine the angular frequency of the vertical oscillations of the disc.

Solution. Let us say that the volume of the liquid in the cylindrical vessel is V_o and that the depth of the liquid would be h_o if the disc were not floating and bobbing in the liquid. Figure 2a

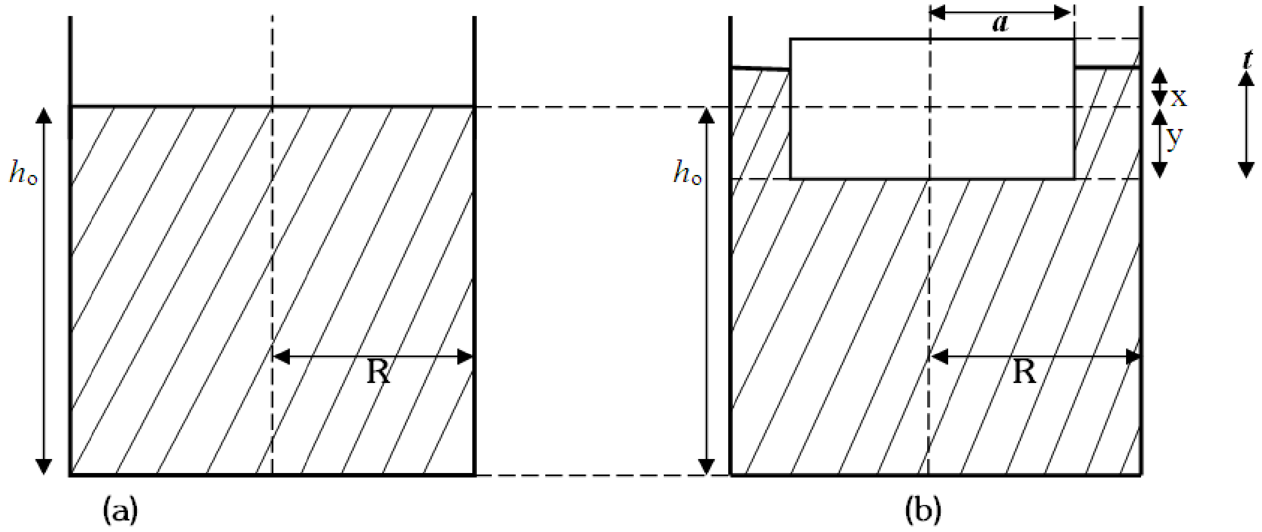


Figure 2. The Geometry of the Bobbing Disc.

shows the cylindrical vessel filled with the liquid to a depth of h_o before the disc is inserted. Figure 2b shows the vessel with the disc displacing a volume of liquid equal to the volume of the part of the disc which is now below the higher surface level of the liquid. We have drawn the Figure as though the central axes of the disc and the cylindrical vessel coincide. As the disc bobs upward, the liquid level descends; as the disc bobs downward, the liquid level rises. As shown in Figure 2b, the horizontal surface of the liquid is x units above h_o and the lower surface of the disc is y units below h_o.

We begin our computations by relating x and y. We note that the volume V of the liquid is not changed by inserting the disc. We may write that V = $R^2 h_o$ (see Figure 2a) and that
V = $\pi R^2(h_o + x) - \pi a^2(x + y)$ (see Figure 2b). Therefore,

$$\pi R^2 h_o = \pi R^2(h_o + x) - \pi a^2(x + y)$$

which implies that $0 = R^2 x - a^2(x + y)$ or

$$x = \frac{a^2 y}{R^2 - a^2}. \quad (2)$$

Next, we analyze the vertical forces acting on the disc and write an equation of motion. The varying upward buoyant force F_o exerted by the liquid on the disc is the weight of the liquid displaced by the disc. That force is $F_B = g\,\rho_{liq}(x + y)\,\pi a^2$ where g is the acceleration due to gravity. The constant downward force acting on the disc is its weight given by $W = g\,\rho_{wood}\,t\pi a^2$.

The magnitude of the unbalanced vertical force acting on the disc is

$$|F_B - W| = \left|\pi a^2 g\left[\rho_{liq}(x + y) - \rho_{wood} t\right]\right|.$$

The mass of the wood is $\rho_{wood}\ a^2 t$. Since y is the displacement of the lower surface of the disc from a fixed level, $\frac{d^2y}{dt^2}$ represents the acceleration of the disc in a fixed frame of reference.

Since the direction of the unbalanced force is obviously opposite to the displacement of the disc, we may now apply Newton's Second Law of Motion to write an equation of motion:

$$\rho_{wood}\pi a^2 t\frac{d^2y}{dt^2} = -\pi a^2 g\left[\rho_{liq}(x+y) - \rho_{wood}t\right].$$

Simplification yields

$$\frac{d^2y}{dt^2} = -\frac{g\rho_{liq}}{t\rho_{wood}}\left[x + y - \left(\frac{\rho_{wood}}{\rho_{liq}}\right)t\right].$$

Recalling that equation (2) gives x as a function of y, we rewrite the equation of motion as

$$\frac{d^2y}{dt^2} = -\frac{g\rho_{liq}}{t\rho_{wood}}\left[\left(\frac{a^2}{R^2-a^2}+1\right)y - \left(\frac{\rho_{wood}}{\rho_{liq}}\right)t\right].$$

Further simplification yields

$$\frac{d^2y}{dt^2} = -\left(\frac{g\rho_{liq}}{t\rho_{wood}}\right)\left(\frac{R^2}{R^2-a^2}\right)\left(y - \left(\frac{t\rho_{wood}}{\rho_{liq}}\right)\left(\frac{R^2-a^2}{R^2}\right)\right). \qquad (3)$$

Finally, we make the change of variable

$$u = y - \left(\frac{t\rho_{wood}}{\rho_{liq}}\right)\left(\frac{R^2-a^2}{R^2}\right)$$

and transform equation (3) into an equation for simple harmonic motion. We now have

$$\frac{d^2u}{dt^2} = -\left(\frac{g\rho_{liq}}{t\rho_{wood}}\right)\left(\frac{R^2}{R^2-a^2}\right)u.$$

The motion is simple harmonic in the variable u with y oscillating about its equilibrium value of $\left(\frac{t\rho_{wood}}{\rho_{liq}}\right)\left(\frac{R^2-a^2}{R^2}\right)$. The desired angular frequency of the oscillation is

$$\omega = \sqrt{\left(\frac{g\rho_{liq}}{t\rho_{wood}}\right)\left(\frac{R^2}{R^2-a^2}\right)}.$$

To get a sense of the numbers involved in our problem, we let the disc bob in water in a cylindrical vessel or radius R = 8 cm. We suppose that the disc is made of white oak and has radius a = 5 cm and thickness t = 4 cm. Water has density 1 gm/cm^3 and the white oak used for the disc has the high density of 0.77 gm/cm^3. We assume that g = 980 cm/sec^2. Evaluating ω for the values given, we find that the angular frequency would be 22.85 sec^{-1} which implies a vibrational frequency of $\frac{\omega}{2\pi}$ = 3.64 sec^{-1}.

<u>Problem 3</u>. A right circular cone with altitude h and volume V has a base of radius r. The vertex angle is θ. The entire mass M of the cone is concentrated at its vertex. The cone is turned so that its vertex points downward and it is then placed in a liquid of density ρ > M/V so that it floats with its altitude aligned with the vertical direction.

The reader may imagine a light conical drinking cup floating in the liquid with a small but heavy metal weight resting at its vertex as suggested by Figure 3.1. We assume that, if the cone bobs up and down, the level of the liquid surface will not be affected and that viscous drag on the cone may be ignored. We also assume that there will be no horizontal perturbations of the cone as it oscillates along the vertical direction.

We ask first for a differential equation of motion for the vertex of the cone as it bobs up and down. Since this equation will turn out to be nonlinear, we also ask for a linear approximation to this equation and for the corresponding simple harmonic frequency for oscillations of very small amplitude.

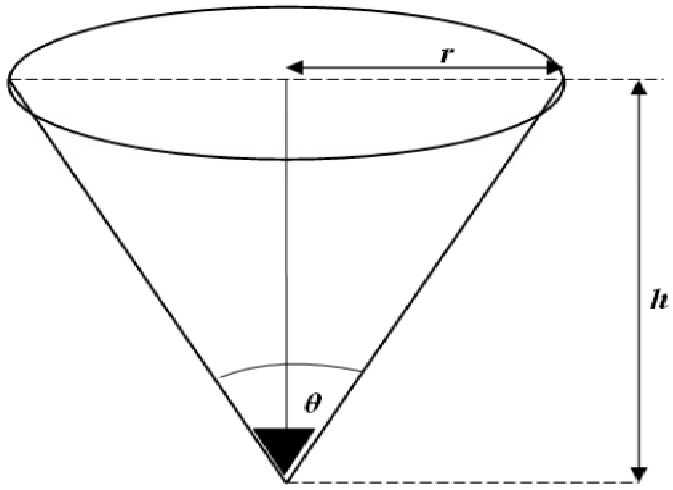


Figure 3.1 The Right Circular Cone.

Solution. We begin by computing the equilibrium depth of the floating cone. We denote the depth of the vertex below the liquid surface by y and the radius of the circular cross section of the cone at the liquid surface by x. We show the geometry of the bobbing cone in Figure 3.2.

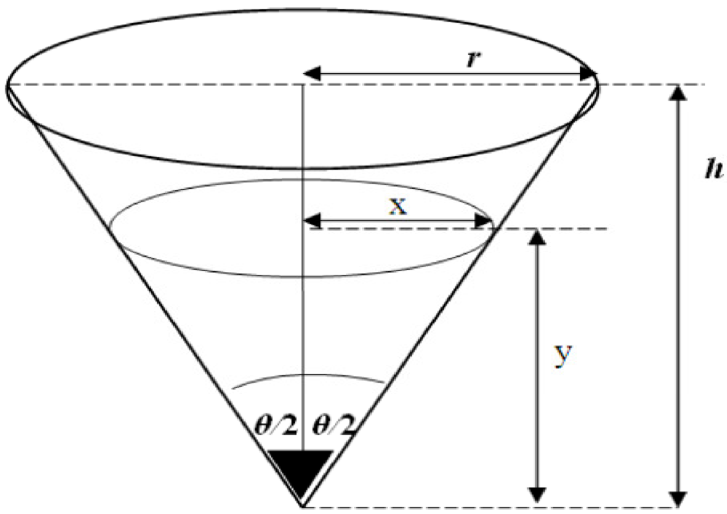


Figure 3.2 The Geometry of the Bobbing Cone.

Let y_o denote the depth of the vertex when the cone is floating at rest in equilibrium. The radius x will have the corresponding value x_o where $\frac{x_0}{y_0} = \tan\frac{\theta}{2}$. Thus $x_0 = y_0 \tan\frac{\theta}{2}$.

At the equilibrium position, the weight of the liquid displaced by the cone must equal the weight of the cone Mg where g is the acceleration due to gravity. Thus

$$Mg = \frac{\pi x_0^2 y_0}{3}\rho g = \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)y_0^3 g}{3}$$

which implies that

$$y_0 = \left[\frac{3M}{\pi\rho\, tan^2\frac{\theta}{2}}\right]^{1/3}. \qquad (4)$$

The last calculations may be repeated in deriving the equations of motion. As the cone bobs up and down, the upward buoyant force is given by

$$F_B = \left(\frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)g}{3}\right)y^3$$

where y varies as opposed to remaining fixed at y_o while the weight of the cone keeps the constant value Mg. Taking the downward direction to be positive for y, the equation of motion becomes

$$M\frac{d^2y}{dt^2} = Mg - \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)gy^3}{3} \qquad (5)$$

Let us recall that in the calculations leading up to equation (4), we showed that $Mg = \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)gy_0^3}{3}$. This recollection allows us to rewrite equation (5) as

$$M\frac{d^2y}{dt^2} = \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)g}{3}(y_0^3 - y^3) = \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)g}{3}(y_0 - y)(y_0^2 + y_o y + y^2).$$

In seeking a simple harmonic approximation of the bobbing cone, we take $|y_o - y|$ to be very small so that $y \approx y_0$. Thus our last equation becomes

$$\frac{d^2y}{dt^2} = \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)g}{3M}(y_0 - y)(3y_0^2) = \frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)gy_0^2}{M}(y_0 - y).$$

Letting $u = y - y_0$, we may write that

$$\frac{d^2u}{dt^2} = -\frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)gy_0^2}{M}u.$$

The angular frequency for simple harmonic approximation is

$$\omega = \frac{1}{2\pi}\sqrt{\frac{\pi\rho\left(tan^2\frac{\theta}{2}\right)gy_0^2}{M}}.$$

Substituting the expression for y_0 given by equation (4) into the equation for ω and simplifying the result yields

$$\omega = \sqrt{g\left(\frac{9\pi\rho tan^2\frac{\theta}{2}}{M}\right)^{\frac{1}{3}}} = \left(\frac{9\pi\rho g^3 tan^2\frac{\theta}{2}}{M}\right)^{\frac{1}{6}}.$$

Then the frequency of vibration for the cone in its simple harmonic approximation is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\left(\frac{9\pi\rho g^3 tan^2\frac{\theta}{2}}{M}\right)^{\frac{1}{6}}.$$

Finally, let us imagine that the cone is bobbing up and down in water. To obtain a feel for the motion for which we have given a mathematical description, let us assign numbers to our symbols and determine the frequency *f*.

The density of water is ρ = 1 gm/cm^3. Let θ = 30°, M = 200 grams, and g = 980 cm/sec^2. We find that f ≈ 2.32 sec^{-1}.

Problem 4. For our Readers

(1992 British Physics Olympiad)

A light pulley of radius R has half of its curved surface covered by a uniform strip of metal sheet of total mass M as indicated in Figure 4. A cord is wound round the pulley and one of its ends is attached to a mass *m* that hangs vertically.

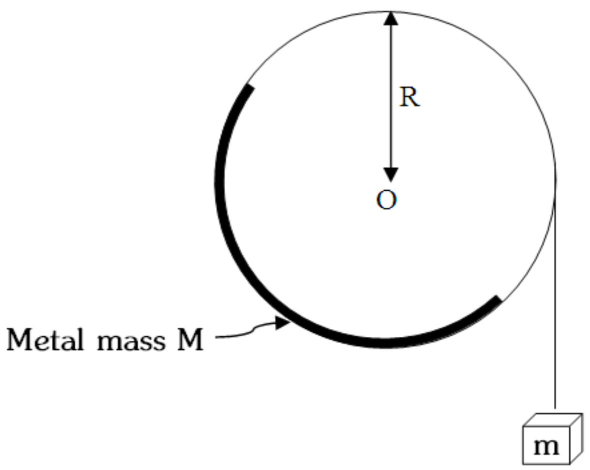


Figure 4. The Pulley with the Strip of Metal and Mass *m* attached to the Cord.

Verify that the distance of the centre of mass X of the metal strip from the axis O of the pulley is $(2R/\pi)$.

Determine each of the following:

The equilibrium inclination, θ, of OX to the vertical for all values of m for which equilibrium is possible.

The values of m for which equilibrium is not possible.

The period of oscillation of the system, for small angular displacements from the stable equilibrium orientation θ, using energy considerations, or the equation of motion.

By means of displacement-time graphs for m, describe its possible modes of motion.

Note that for small $\alpha, \cos(\theta + \alpha) = \cos\theta - \alpha\cos\theta - \frac{\alpha^2}{2}\cos\theta + \cdots$

Physics Education and PYPT

Dina Izadi

Ariaian Young Innovative Minds Institute (AYIMI), Tehran, Iran
http://www.ayimi.org, info@ayimi.org

Abstract
It has been recognized that physics learning and teaching should change from traditional to research method and findings from Physics Education Research (PER) had initiated a tremendous effort in improving physics teaching. New methods indicate that student understanding and misconceptions from quantitative interpretation research has built a pathway for curriculum development and effective teaching of introductory physics. Active learning is a method which has caused to increase student conceptual understanding. This encourages students to construct their own knowledge. There is increasing evidence that many students are unable to apply the physics that they have studied. For meaningful learning to occur, students need more assistance than they can obtain through listening to lectures, reading the textbook, and solving standard quantitative problems. Substitution the traditional models of teaching and learning physics with new one will promote active learning. Different national and international tournaments which are either valuable to students or educators will improve their skills. Persian Young Physicists' Tournament, PYPT, is one of the programs which Iran has involved since 2007. This tournament provides practice in interpreting various representations, e.g., formulas, graphs, diagrams, and verbal descriptions. Students solve the problems by research which is one of the most important parameters in physics education. PYPT workshops are another opportunity for students who have been in previous PYPTs to educate other students and learn how to be a teacher. In this case students will be trained to become future scientists. Ariaian Young Innovative Minds Institute (AYIMI) is the organizer of PYPT in Iran which holds this tournament with cooperation of universities and organizations each year to select team for International Young Physicists' Tournament (IYPT).

1. Introduction

Scientific laws, principles and concepts can be better understood and experienced by the interactions of individuals solving problems in groups and in a laboratory style and the correlation between algorithmic mathematical skills and problem solving in physics.

During the last 15 years, in most countries, the popularity of physics among students has been low and the enrolment has declined so different ways instead of traditional physical science instruction will lead to improve physics education for all students.

In active learning, Studio Model will help students work in a group and do their experiments to get the physics laws and teachers just help them with the experiments, arranging discussion, and guiding students toward a correct conclusion. Discovery Lab will provide an opportunity for students to carry out experiments with mathematical expressions. They experiment to collect evidence and draw a conclusion based on that evidence.

Most students like novices see physics more as isolated pieces of information unrelated to the real world. But what we specifically should do students think like an expert. They should learn physics by experiments and see the content of physics as the concepts that describe nature and use in a wide variety of situations. Students should be able to solve problems correctly by learning the useful concept not by memorizing. So instructors can move students from mindless memorization to understanding. If teachers try to teach physics in the context of everyday life applications, students are more likely to recognize other applications where physics enters their daily lives. To engage students through interactive exploration of the physics and creation of fun, challenges are one the most important factors can impact their learning and attitudes towards physics.

International Young Physicists' Tournament, IYPT, and Persian Young Physicists' Tournament, PYPT, are new methods in physics education which students know and understand definitions, terminology, facts, concepts, principles and operations. They are able to communicate what they know with others and know how to apply what they have learned to analyze situations and solve problems; and can develop their ability to evaluate critically the usefulness of various problem-solving approaches.

2. PYPT in Education

To provide a program of educating and supporting teaching assistants, Persian Young Physicists' Tournament, PYPT, has been organized to develop assessment tools to evaluate student progress in problem solving, technical presentation in English which is not their native language, and team working. The 1st Persian Young Physicists' Tournament (PYPT) was in March 2008 which two selected teams participated in Austria (AYPT) and IYPT to get the first experience from this attractive challenge. Now students from different high schools in Iran are able to request entry into PYPT which is carried out in a period determined by the PYPT Executive Committee (ECO). The best teams challenge in final and receive rewards but the best students with highest individually scores as the PYPT Regulations are selected and after education, participate in International Young Physicists' Tournament, IYPT. The rules for presentation of the results, opposition, reviewing and judgment by the jury have been fixed in the Regulations of PYPT which is different from regulations of IYPT in some parts. PYPT Juror are selected from different universities such as University of Tehran, Amirkabir University of Technology, Sharif University of Technology, Alzahra University, Islamic Azad University, Tarbiat Modares Uniersity,…each year

There are three students in a group, from upper secondary schools, up to the age of 18 or 19, which are able to request entry into PYPT. They should work on IYPT problems which is on the PYPT website too (http://pyptonline.com). There are not specific solutions for these problems so research and experiment are the main factors to direct them in finding the best solution.

PYPT workshop is the other educational community which plays an important role in directing Jurors and students who need to learn more about PYPT regulations.

3. PYPT problems

There are 17 problems should be solved by each team but five problems can be rejected in PYPT without penalty. In solving these problems students learn how to research, work in lab, share their findings in their group and compare their results in experiments with principles and physics' laws. Some parts of three problems as follow define how students work to present in an international tournament.

- *Magnetic Spring (Problem has been solved by: Reza Montazeri Namin)*

Two magnets are arranged on top of each other such that one of them is fixed and the other one can move vertically. Investigate oscillations of the magnet.

The theory to solve this problem has two main parts: a) To find the force that the magnets exert to each other in different distances, b) Using the force, to find the equations of motion of the top magnet. Finding the force may be done using two main models: The Gilbert's model, which considers each magnet to be made of two poles, and every pole acts like an electric pole. The Ampere's model, which considers each magnet to be a solenoid, and due to the current in the solenoids, the magnets exert forces to each other. After finding the force that the magnets exert to each other, we can find the equations of motion and oscillations of the top magnet. We know that there are three forces exerted to the top magnet: Gravity, magnetic force & friction. So we have all the forces that are exerted to the magnet. But again, because of the complicated form of the magnetic force, the differential equations of motion will not be solvable, so again we use a numerical method to investigate the movements.

Similar to the theories, the experiments have two main parts: Checking the force to decide the correct theoretical model, and investigating the oscillations and comparing with the theories. The force evaluation was done by adding weights on the top magnet. By adding weights, the distance between the magnets would change. While we know the mass of the magnet and the weights, we could find the force that the magnets exert to each other in different distances (Fig. 1).

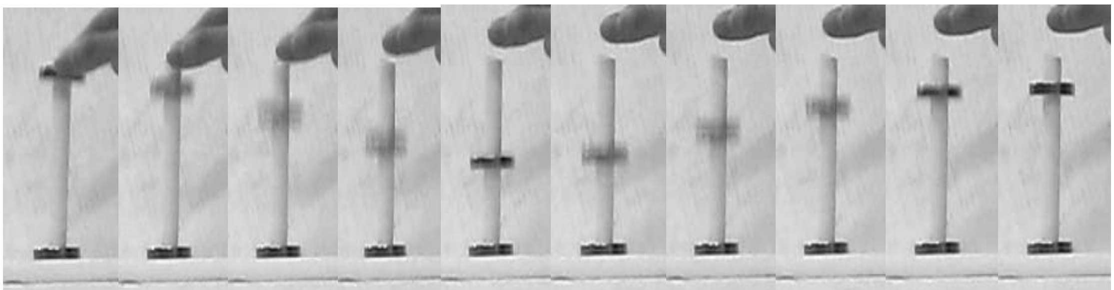


Fig. 1. Oscillations of a magnet in different distances

Shrieking rod (Problem has been solved by: Zahra Karimi)

A metal rod is held between two fingers and hit. Investigate how the sound produced depends on the position of holding and hitting the rod?

Working with the sound waves while there are several types of motion and several patterns, can be a complicated task. It's been proved that both longitudinal and transverse waves can propagate in solids.

In the case of cylindrical metal rod which should be held at a certain position within its length and be hit, for finding the right patterns of propagation paying attention to both holding point and hitting type is necessary.

Holding points can be considered as nodes because approximately there is no motion at these points, these nodes are really critical in understanding the accurate pattern of longitudinal and transverse waves. A metallic rod is quite similar to a tube which is open from both ends so when it comes to longitudinal motion only the fundamental frequency or harmonics which have the same place of nodes with the holding point can be observed. this means that the longitudinal wave can only be produced by holding the rod at even multiples of the length (1/2L.1/4L.1/6L,...) while there are transverse waves caused by vibration from the hitting all the time.(Fig. 2)

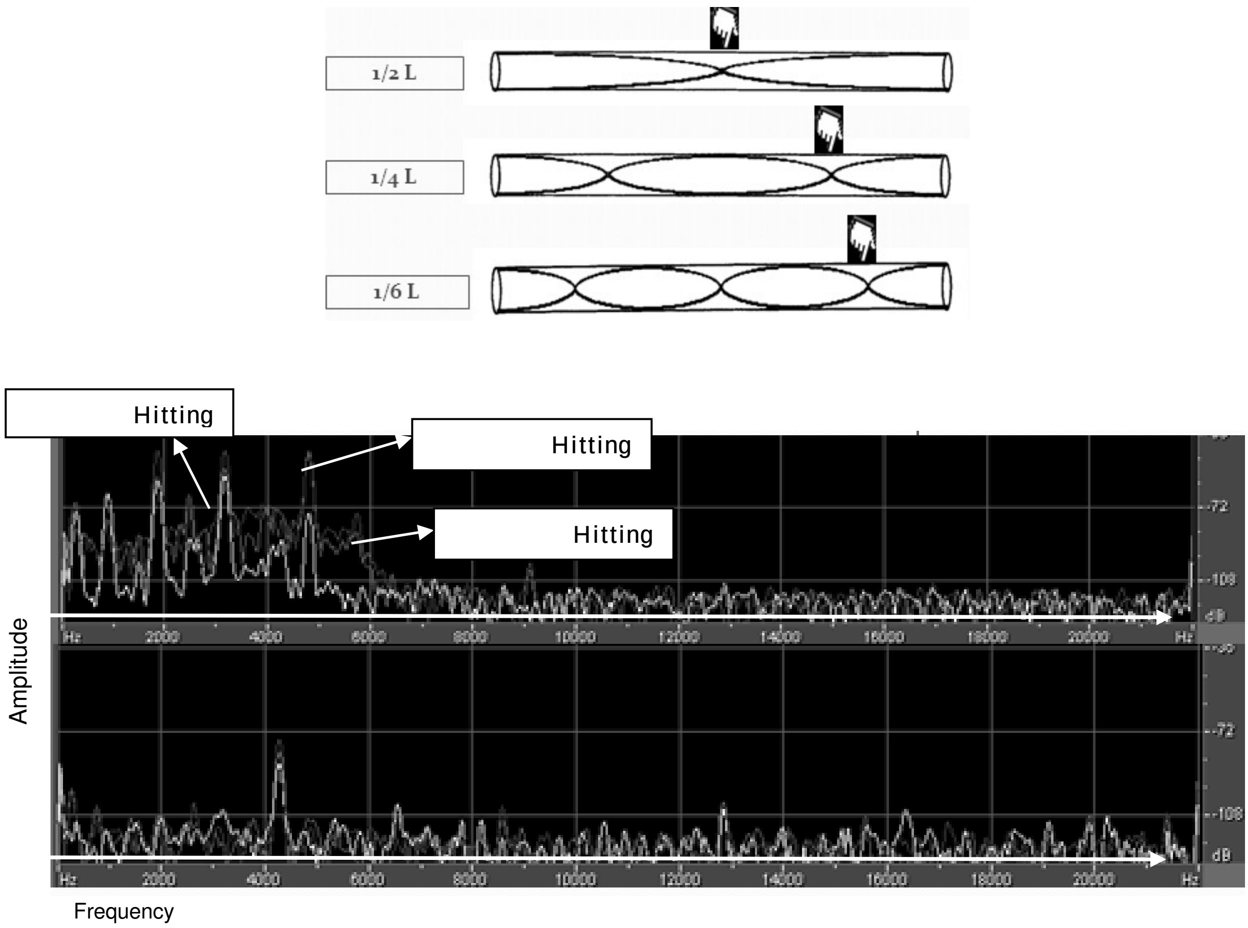


Fig. 2. Transverse waves cause by vibration from the hitting

- *Electromagnetic Cannon (Problem has been solved by: Saba Zargham and Hamid Ghaednia)*

A solenoid can be used to fire a small ball. A capacitor is used to energize the solenoid coil. Build a device with a capacitor charged to a maximum 50V. Investigate the relevant parameters and maximize the speed of the ball.

As we know, a solenoid consists of helical connected coils. When a current is sent through the solenoid, it generates a magnetic field, which its magnitude can be defined by the Bio-Savart law. This magnetic field can exert force on conductive material. Ferromagnetic materials are made up of many magnetic dipoles.(seen as current loops) When placed in a magnetic field, these dipoles tend to align with the field. Due to this alignment a current is produced throughout the matter. This force can therefore by calculated by the Lorentz force. And it is the same force used to build the electromagnetic cannon based on the solenoids principle.

The solenoid is wrapped around a diamagnetic_hallow cylinder. Depending on the shape of the coil, a plunger (that consists of two parts, a ferromagnetic part and a non-ferromagnetic part which is the shooter), can be situated in the solenoid in a way that can move in and out of the center. The plunger is used to provide mechanical force which will be used to kick the ball (Fig. 4).

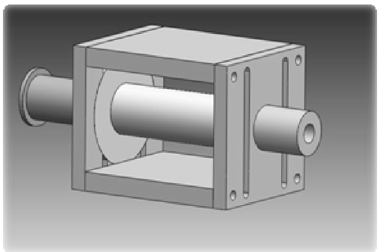


Fig. 4. The plunger used to provide mechanical force in electromagnetic cannon

The final speed of the ball is much related to the force exerted to it by the plunger which itself is only determined by the force applied to the plunger by the solenoid. Therefore calculating this force is necessary. To do so, we have first calculated the force applied to circular-shaped element_of the solenoid from a distance r. (the force along the x axis is the one important for us) and so by integration we're able to determine the net force applied to the plunger (Fig. 5).

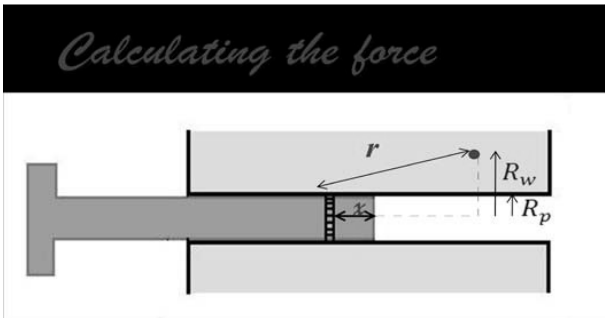


Fig. 5. Force calculating in Electromagnetic cannon

$$dF_x = i\left|dl_{plunger} \times B\right| csc\theta$$
$$dl = Rd\theta$$

(The full papers have been published in IYPT 2010-2011 Proceeding).

4. Statistical Survey

The number of participants in PYPT has increased in recent years in comparison with last PYPTs (Figs. 6). Students are girls and boys from different schools which during the period of training, their teachers or team leaders educate them practically. young students talk about the conceptual approaches which help them in solving problems. Combination of the knowledge and experiments to motivate students is an important factor in active learning but to improve it in a high level we need a debating and asking in a cooperative atmosphere.

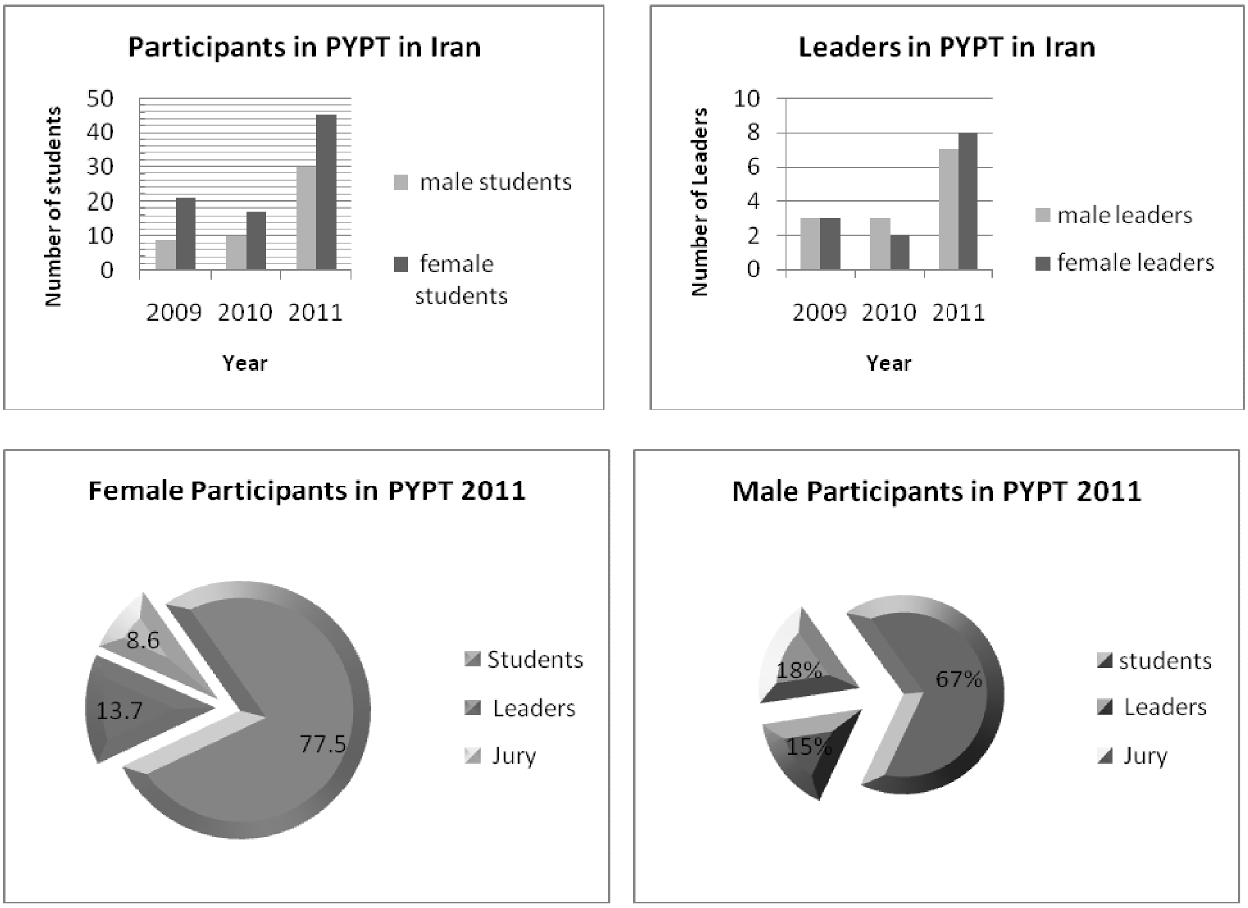


Fig. 6. Statistical survey in PYPT

5. Conclusions

In spite of the best efforts of teachers, typical students are also learning that physics is boring and irrelevant to understanding the world around them. Many college teachers today want to move past passive learning to active learning, to find better ways of engaging students in the learning process. But many teachers feel a need for help in imagining what to do, in or out of class that would constitute a meaningful set of active learning activities. So we need to change science education to make it more attractive and relevant for a much larger fraction of the student population than in the past. PYPT and IYPT is a new method in physics education which lead to greater retention of knowledge, deeper understanding, and more positive attitudes toward the subject being taught.

References

1-Izadi D., Torabi A. M., Mahmoudi N.,Izadipanah N.,Eshghi N., "Reports on Activities in Bssic Sciences in the Last Three Years in Iran", 4th ICWIP conference, South Africa, April 5-8, 2011.

2-http://www.pyptonline.com

3-http://www.ayimi.org

4-http://phet.colorado.edu

5-http://www.phys.washington.edu

6-http://groups.physics.umn.edu

7- Carl Wieman and Katherine Perkins, "Transforming Physics Education", Physics Today, November 2005, 58(11), p.36

8- Ian D. Beatty Æ William J. Gerace, "Technology-Enhanced Formative Assessment: A Research-Based Pedagogy for Teaching Science with Classroom Response Technology", Published online: 7 January 2009

10- Paosawatyanyong B, P. Wattanakasiwich, " Implication of physics active-learning in Asia", *Lat. Am. J. Phys. Educ. Vol. 4, No. 3, Sept. 2010*

Operating Temperature and Heat Capacity of a Light Bulb Filament
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An Experimental Problem used in the German Physics Olympiad

[1] Axel Boeltzig, [2] Stefan Petersen

[1] Institute of Nuclear and Particle Physics (IKTP), TU Dresden, Germany
[2] Leibniz Institute for Science and Mathematics Education (IPN), University of Kiel, Germany

Abstract
We present an experimental problem used in the German national competition for the International Physics Olympiad, which, due to its emphasis on accurate data acquisition and rather unusual ways of evaluating the data, seems very suitable for testing the experimental abilities and creativity of the students.
The first part of the problem deals with the resistance of a light bulb filament as a function of temperature. The determination of the filament's temperature utilizes features of its black body radiation curve and the wavelength-dependent sensitivity of a photo diode. The analysis with the given instruments requires clever application of a logarithmic plot.
In the second task the heat capacity of the light bulb filament is investigated. This can be done by periodic heating and cooling of the filament. The process is driven by an appropriate voltage signal from a function generator and monitored using an oscilloscope. This task can also be used as a single problem, when the result of the first part is given.

Context
The selection competition for the German IPhO-team, the German Physics Olympiad, is organized by the Leibniz Institute for Science and Mathematics Education (IPN) at the University of Kiel and consists of four stages all carried out at a national level. In the first two stages the students solve sets of given problems at home. In each of the final two rounds the best participants come together and face two theoretical as well as two experimental exams, each lasting three to four hours.
The presented experimental problem is in parts based on an idea described by Kraftmakher (Kraftmakher 2004) and was posed for an exam in the fourth and final round of the competition in 2011. The 15 participating students had four hours time to work on it.

Preparation of the students
The second task involves an analogue oscilloscope and a function generator. To ensure that the participants could focus on physics rather than pure handling of the devices, an introduction on the equipment used was given two days before the exam.

For a hands-on training the group of students was split into pairs. One student was asked to set the function generator to given parameters, the other one then asked to find out these parameters by analyzing the signal with the oscilloscope. The roles were changed afterwards.
Special attention was paid to aspects that were considered potentially important for the problem. Some signals were similar to those that could have been used for the second task.

Problem

Introduction

The predominant process for heat transfer at high temperatures is by radiation. In this experiment you are asked to examine a filament of a light bulb at high temperatures.

Material

- Opaque box with four connectors, containing:
  - a light bulb with a nominal voltage of 6.0 V (maximum voltage 8.0 V)
  - a photo diode (Osram Components BPW 34)
- 9 V battery with a resistor of 100 Ω connected in series
- Power supply (variable DC voltage)
- 3 multimeters
- Function generator
- Oscilloscope
- Connectors and cables
- (10 0.1) Ω resistor
- 4 transparencies with graphs (see problem text and Figure 3)
- Graph paper, ruler, marker, pencil, eraser, adhesive tape

The uncertainty of the multimeters can be estimated as 2 % of the set measuring range. A list of their inner resistances is available. The ground contacts of oscilloscope and function generator have the same potential. Avoid short circuits!

Properties and use of the photo diode

The opaque box with the light bulb holds a photo diode. Using this diode, the light intensity of the bulb can be measured. When a constant voltage is applied to the diode in reverse direction, it acts as a current source with an output current proportional to the power of the incident radiation.

However, the constant of proportionality between incident power and output current depends on the wavelength of the light. For the diode in the box, this relative sensitivity is shown in Figure 1 (the data was taken from the datasheet mentioned in the references).

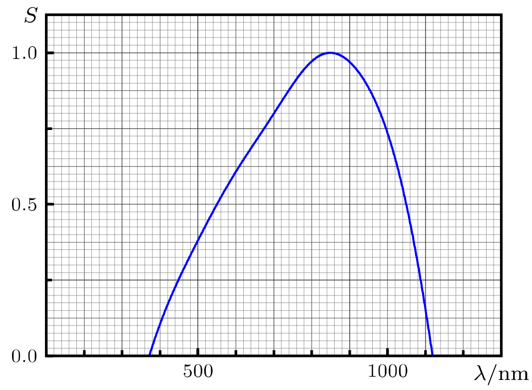

Figure 1: Relative sensitivity of the photo diode as function of the wavelength of the incident light.

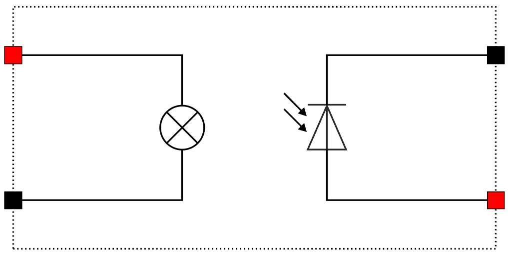

Figure 2: Illustration of the inner box layout as printed on the box given to the students.

Task 1

The graphs on the given transparencies (see Figure 3) show the current of such a photo diode when it is exposed to the radiation of a black body with different temperatures T, with no other sources of radiation present and, except for T all conditions left constant. The graphs differ only in the ways their axes are scaled. Note that the current signal is given in arbitrary units.

a) Give a brief and qualitative description, how the graphs on the transparencies can be calculated using the graph from Figure 1.

b) Examine the intensity of the light bulb for different voltages applied to it, in order to determine the electric resistance of the light bulb at different temperatures of the filament.
Plot the electric resistance R_B of the bulb against its temperature T_B and compare your result to a linear dependence of R_B on T_B.

c) Determine the temperature T_N of the light bulb filament at nominal voltage.

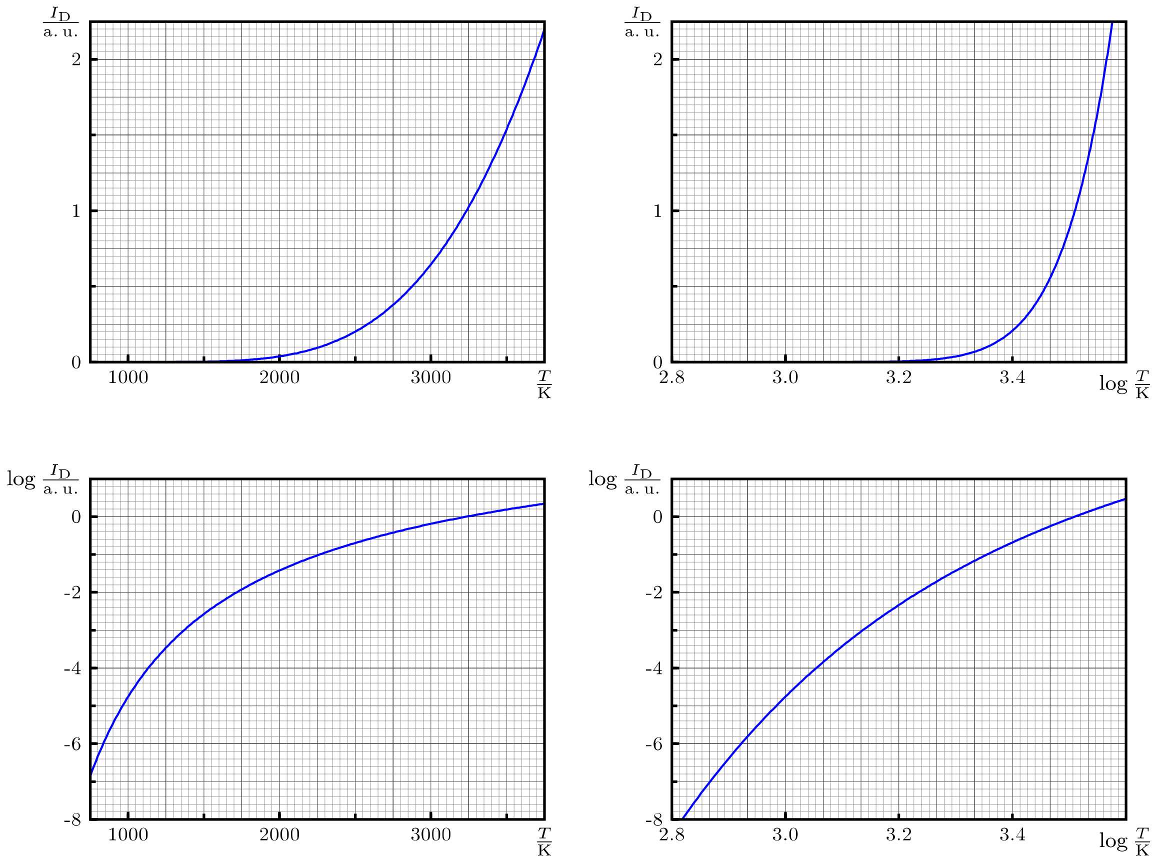

Figure 3: The plots of $I_D(T)$ with different scales as given on the transparencies for the students. Each plot for the participants was on an A4 transparency.

Task 2

The heat capacity of the filament can be determined with the given devices. To this end the light bulb can, for instance, be operated by square waves of low frequency produced by the function generator. The oscilloscope can be used to observe voltage curves.

Determine the heat capacity C of the filament. Name all approximations made, and the conditions for your result to be valid.

Solution to the experimental problem

Task 1

The fact that the photo diode's sensitivity depends on the wavelength of the incident radiation is crucial for this problem. Although the radiation power is proportional to T^4 according to Stefan-Boltzmann law, the sensitivity's dependence on the wavelength yields a different relation for the diode signal.

The output current I_D of the photo diode is proportional to the power of the incident light, weighted by the diode's spectral sensitivity $S(\lambda)$:

$$I_D \propto \int_0^\infty \frac{dE}{d\lambda}(T) \cdot S(\lambda) \cdot d\lambda$$

The spectral distribution $dE/d\lambda$ of the filament's radiation power is given by Planck's law. This integral can be evaluated numerically, which yields the given graphs. The current can only be given in arbitrary units, since the factor of proportionality is not known in this case.

As stated in the problem text, radiation is the dominant way of losing heat for the filament. Other means of heat transport can be neglected. This is also true for the absorption of radiation from the environment.
When the light bulb is operated at a constant voltage U_B with an according current I_B, a thermal equilibrium is quickly established. Thus the bulb's temperature can be derived from its power consumption, except for an unknown factor, here called α:

$$\sqrt[4]{I_B \cdot U_B} = \alpha \cdot T_B$$

In addition to the values of U_B and I_B, the current signal I_D of the photo diode can be measured for each data point.
The expected temperature dependence of I_D, given in the graphs of Figure 3, was discussed above. But since the theory only yields a proportionality, there is another unknown factor between the measured current and the numerical value given in the graph, here noted as β.
So instead of I_D over T_B as given in the graphs, the measurements only yield $\beta \cdot I_D$ over $\alpha \cdot T_B$. However, when plotting $\log(\beta \cdot I_D)$ over $\log(\alpha \cdot T_B)$ one obtains the same shape of the curve, while the coefficients α and β are merely reflected as shifts in x - and y - direction.
Therefore one can match the given logarithmic curve and the measured data by placing the transparency on top of a graph of the measured values. The same scales have to be used in both plots and the transparency may not be rotated, of course. Figure 4 shows the result with the data points shifted onto the given graph from Figure 3. The error bars are left out for reasons of clarity, which is justified since the errors become very small at higher temperatures and higher diode currents.
The value of α can be derived from the shift in x - direction and subsequently be used to determine the temperature T_B of the filament from its electrical power consumption in thermal equilibrium. The uncertainty of α can be estimated by variation of the shift. The data aligns well with the full-sized plots for a shift of about ± 2 mm, which is equivalent to a relative uncertainty of about 1.5 % for α. For the nominal voltage U_N one obtains a filament temperature of about $T_N = (2800 \pm 70)K$.
The plot of the bulb's resistance R_B as a function of T_B in Figure 5 shows very good linearity, with the resistance of the bulb approximately given by $R_B(T_B) = 22.9\frac{m\Omega}{K} \cdot T_B - 6.64\Omega$. Note that the linearity could be checked even without knowledge of α. The obtained relation $R_B(T_B)$ will prove to be useful in the next task.

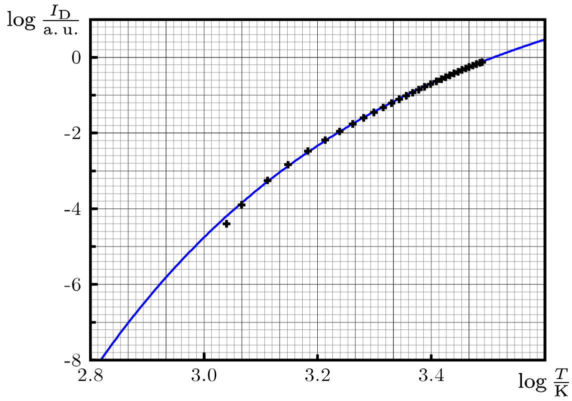

Figure 4: Experimental data points shifted onto the logarithmic plot of Figure 3.

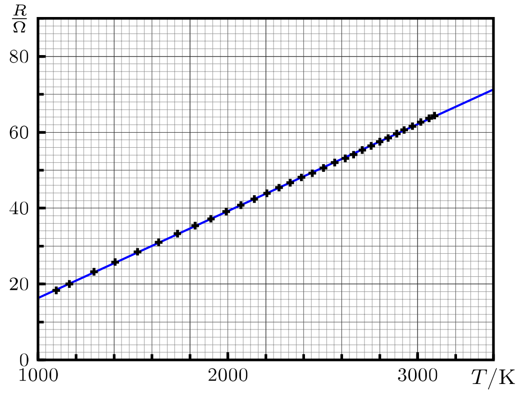

Figure 5: The results for $R_B(T)$ and a fitted linear function.

Task 2

The focus for the students' solutions for this task was on creative application of the given devices to determine the heat capacity, rather than obtaining a high accuracy. However, the students were asked to be aware of any assumptions or simplifications of their method, and to name them.

The heat capacity is relevant for processes out of thermal equilibrium, such as heating of the filament after switch-on, or cooling after switch-off. Since heating and cooling in the relevant high temperature range happens on rather short time scales (typically between 10 and 100 ms), the oscilloscope has to be used to study these processes. As the given oscilloscope does not provide a storage function, the observed processes are required to run periodically.

There are different ways to determine the heat capacity. We will present two of them here.

As suggested in the problem text, one can periodically switch the light bulb on and off using the function generator. This can be achieved with a square wave of amplitude u' and an offset of the same value. This means that the bulb is switched on with a voltage of $2 \cdot u'$ for the first part of the cycle and switched off the other time. The duration and relation of these periods can be changed with different frequencies and duty cycles of the signal.

To examine the light bulb, its voltage $U_B(t)$ and current $I_B(t)$ need to be measured. The voltage $U_0(t)$ supplied by the signal generator can directly be displayed on the oscilloscope. To measure the current, the given resistor can be connected in series to the bulb, with the voltage $U_R(t)$ over the resistor shown on the oscilloscope.

With this setup, the bulb's voltage and current can easily be derived from the observed quantities $U_0(t)$ and $U_R(t)$ using the following relations:

$$U_0(t) = U_B(t) + U_R(t)$$

$$U_R(t) = R \cdot I_R(t) = R \cdot I_B(t)$$

Note that the electric power $I_B(t) \cdot U_B(t)$ can not be used to determine the filament's temperature $T_B(t)$ here, as no thermal equilibrium is established. However, inverting $R_B(T_B)$ obtained in the first task, the resistance $R_B(t)$ can be used for that purpose.

One way to obtain the heat capacity is to directly determine the heat absorbed by the filament during a certain time interval of the heating period:

$$q = \int_{t1}^{t2} (I_B(t) \cdot U_B(t) - \alpha^4 \cdot T_B(t)^4) dt$$

The heat capacity then is:

$$C = \frac{q}{T_B(t_2) - T_B(t_1)}$$

This method requires careful and detailed recording of the curves to obtain good results.

A different and more practical approach is to consider the cooling intervals when the light bulb is switched off. One can determine the temperatures $T_B(t_1)$ at the end of a heating period and $T_B(t_2)$ at the beginning of the next one. In between the filament cools due to radiation:

$$C \cdot \dot{T}_B(t) = -\alpha^4 \cdot T_B(t)^4$$

After integration this yields:

$$\frac{1}{3 \cdot T_B(t_2)^3} - \frac{1}{3 \cdot T_B(t_1)^3} = \frac{\alpha^4}{C} \cdot (t_2 - t_1)$$

With different amplitudes, duty cycles and frequencies of the supply voltage, the initial temperature and the duration of the cooling period can be altered. The formula above suggests a plot of the left hand side expression as a function of the cooling time $\mathrm{t}_2 - \mathrm{t}_1$ as shown in Figure 6.

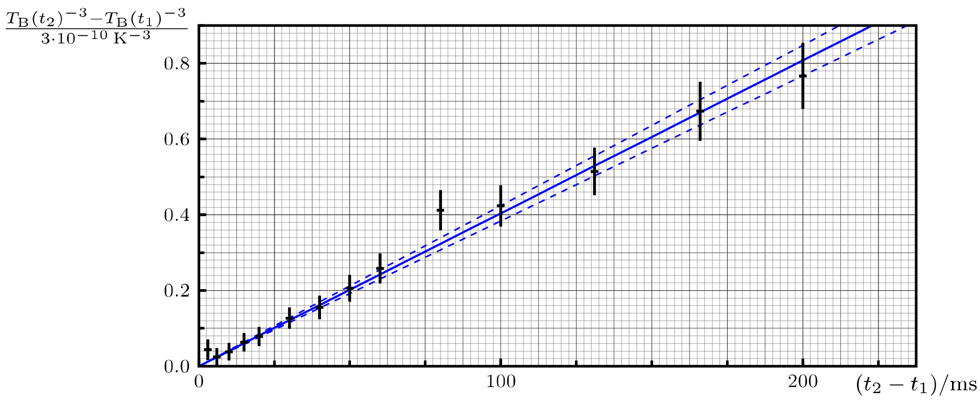

Figure 6: Plot to determine the heat capacity from the cooling phase of the filament. The dashed lines represent a 5 % variation of the fitted curve's slope.

Different measured quantities and previous results are used to calculate T_B. Accordingly the uncertainty of the plotted left hand side expression shown in Figure 6 is determined by many factors. However, the main contribution results from the two voltage readings on the analogue oscilloscope. The relative uncertainty of the time difference on the abscissa is negligible.

With the tools available during the exam, the uncertainty of the slope can only be estimated. A relative uncertainty of 5 %, as indicated in Figure 6, appears to be justified. For the calculation of C, the uncertainty of α discussed before also has to be considered.

Using the value of α determined before and the slope of the linear fit yields the heat capacity with $C = (2.2 \pm 0.2) \cdot 10^{-5} \frac{J}{K}$.

In addition to the uncertainty of the result it is also important to recognize the assumptions used in the presented techniques to determine C. For the two methods, the heat capacity was assumed to be constant in the examined temperature intervals. This is an approximation, as the specific heat capacity of the material may vary in the rather large temperature range. The specific heat capacity of tungsten for example increases by about 20 % in the temperature range from 1500 K to 2500 K according to (White 1997). Additionally the possibility of other parts than the filament changing temperature (thus contributing to the heat capacity) was neglected.

Within the scope of this exam, the students could not be expected to test these assumptions, but should be aware of them.

Conclusions

The two experimental tasks, albeit considered very interesting by the students, proved to be too extensive for the given amount of time. Even though roughly two thirds of the 15 candidates were able to achieve reasonable results in the first tasks only two of them received close to full marks. The second task was more or less successfully solved by only one of the students. These results can to some extent be attributed to the lack of time but may also be an effect of the unusual format of the experimental problem.

Altogether it can be said that the students certainly profited from the intensive training on the equipment used and that they were rather motivated to work on the problem. For the purpose of differentiating between the students in the competition the problem could have been shortened considerably, e.g. to only one of the two tasks.

Acknowledgements

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References

Kraftmakher, Y. (2004). Pulse calorimetry with a light bulb. *European Journal of Physics*,25(6):707-715.

White, G. K. and Minges, M. L. (1997). Thermophysical Properties of Some Key Solids: An Update. *International Journal of Thermophysics*, 18(5):1269-1327.

Datasheet of photodiode of type Osram Components BPW 34. http://catalog.osram-os.com/catalogue/catalogue.do;jsessionid=?act=downloadFile&favOid=020000030000c226000100b6 (08.11.2011).

A few good orbits

Chiraag Juvekar[1], Mehul Jain[1] and Aniket Sule[2,#]

[1]Indian Institute of Technology (Bombay), Mumbai, Maharashtra, India - 400 076.
[2]Homi Bhabha Centre for Science Education (HBCSE). Mumbai, Maharshtra,India - 400 088.
[#]Corresponding author: anikets@hbcse.tifr.res.in

Abstract
We present three novel problems based on Kepler's laws, which help in bringing out various facets of orbital dynamics. The problems were designed by the authors for the training and selection camp of the Indian National Astronomy Olympiad 2008. The problems require only pre-calculus mathematics and high-school physics, but demand strong visualization of the scenarios presented. By constructing fun problems involving elaborate situations like the ones presented here, students are exposed to practical applications of the Kepler's Laws like satellite orbits.

1. Introduction

The Indian National Astronomy Olympiad program started in the year 1999. The Olympiad competitions typically involve pre-university students who compete through various stages finally leading to selection of national team for respective international Olympiads. Typical enrollment for the first stage of Astronomy Olympiad in India is close to 15,000 students. As pre-university curriculum in India does not include advanced topics in astronomy and astrophysics, initial selection tests focus on the students' understanding of physics, mathematics (pre-calculus) and general mental ability. The last stage of selection is a 3 week summer camp for nationally selected top 50 students who are exposed to various topics in astronomy and astrophysics, involving nontrivial physical arguments and pre-calculus mathematics. Designing problems for the selection tests during the camp is a challenging task as the problems are expected to be unconventional and yet within boundaries of student's scholastic competencies. Over the years a number of fairly sophisticated problems in various fields of astronomy have been posed in the various levels of the Olympiad program leading up to the selection of the national team. To level the playing field for all participants who may have prepared in different ways prior to the selection camp, all problems in selection tests are newly designed (i.e. none of them are pre-published) by the Olympiad resource persons, who include teaching faculty as well as past Olympiad medalists. What follows is an analysis of three problems designed by the authors in the field of orbital dynamics which were posed in the 10th Indian National Astronomy Olympiad camp in the year 2008.

2. A Brief Overview of Orbital Mechanics

The basic orbital mechanics is governed by just two equations: Newton's law of gravitation and Newton's second law of motion. The first provides an approximate expression for the force due to gravitation between two bodies. The second relates this force to the acceleration of the body enabling us to solve the kinematics of the bodies involved. Based on these two equations, a complete solution for the two body problem is well-known as a one-body problem with a reduced mass plus the free motion of the center of the mass. We can approximate the reduced mass to the mass of the bigger body (called the central body), if it is many times as massive as the second (orbiting) body. For this special case, lets us summarize necessary terminologies, laws etc.

1. Kepler's Laws:
    a. All orbits are conic sections with the central body at one of the foci (let us call this as the prime focus).
    b. The area of the sector, centered at the prime focus, traced by the orbiting body is proportional to the time taken to cover it.
    c. The square of the time period of the orbiting body is proportional to the cube of the semi-major axis of its elliptical orbit.

2. Peri- and Ap- position: At some point in its orbit the orbiting body must be at its closest distance to the central body. This is peri-position. Likewise, the furthest distance is the ap-position. Obviously for unbound obits the ap-position does not exist (i.e. it is at infinity).
3. True-anomaly(V): The angle subtended by the radius vectors from the prime focus to the peri-position and the current position of the orbiting body.

The understanding of Kepler's Laws and teaching the topic in an elegant manner has been an important topic of discussion amongst physics education community [1], [2], [3], [4], [5], [6], [7], [9][10].

3. The Solution and the Analysis of the problems

What follows are the actual statements of the problems and their solutions.

3.1. Problem 1 : Celestial Spheres for the Extra-Terrestrial

3.1.1. Problem Statement:

4N1K37-2 is launched from the Earth into a circular geosynchronous orbit and further it is ensured that the phase of the Earth in the space shuttle's sky remains constant The space shuttle's transmission antenna always points towards the Earth and the solar panels always face the sun. The solar panels were built with an inherent gravitational field such that people can stand on them. The space shuttle began its transmission to the ground station (somewhere on the Equator) when it was in the Equatorial Plane of the Earth on summer solstice day.

At the beginning of the transmission, an observer Alice who is at the ground station could see the space shuttle on her zenith and the sun on her western horizon. Suppose an observer, Bob, standing on the solar panel of the space shuttle is facing the Earth. Find the constellation in the anti-Earth direction of the observer (on the space shuttle) after exactly 100 (sidereal) days of transmission.

3.1.2. Motivation:

It is often possible to confuse between a geostationary and a geosynchronous orbit. In the case of a geo-synchronous satellite all that is required is for the period to be equal to one day. In other words, for a fixed observer on the ground, it should return to the same position in the sky at the same time every day. The geo-stationary orbit is special case of the geo-synchronous orbits, where the orbit is equatorial and circular. More importantly, the problem demands strong spatio-visual thinking that is generally neglected in science teaching, especially in India.

3.1.3. Solution:

The problem inherently is very simple. All that is needed is a clear mental picture. This is the step by step procedure to identify the orbit:

i. Firstly we note that the orbit is a geosynchronous orbit. That means, after 24 hrs the satellite returns to the same position relative to the earth. Thus the length of the semi-major axis (a) of the orbit is fixed.
ii. The orbit is given to be circular. Hence the eccentricity (e) is also fixed to be 0.
iii. Next we find the inclination (i). It is given that in the satellite sky, Earth has constant phase. If the Sun-Earth-Satellite angle is θ at a given moment that after 12 hours it will be $(180° - \theta)$. Thus the phase angle would change unless of course θ were maintained to be 90°. The orbit which does this is the one perpendicular to the ecliptic and parallel to the terminator.
iv. Now, we must find the initial position in the orbit. It is given that the first transmission occurs on summer solstice day. Thus the Earth-Sun vector points to an ecliptic longitude of 90°. Now the satellite can have 2 positions in this orbit. One such that the Earth-Satellite vector point to a longitude of 0° and the other such that it points towards 180°. The fact that observer Alice saw the satellite at her zenith and also saw the Sun in the western sky at the same time implies that the Earth satellite angle is in fact 180° ecliptic longitude.
v. As for the direction of revolution we could have either but the fact that the second position is asked after exactly 100 sidereal days renders the point irrelevant.

Now we note that initially the Anti-Earth direction was the autumnal equinox point. After 100 days which correspond to approximately 3 and $\frac{1}{3}$ months, the constellation will shift by three zodiac constellations. Initially the anti-Earth side was Virgo and finally it turns out to be Sagittarius. (Note: Rough Calculations show that the ecliptic longitude turns out to be 278)

3.1.4. Remarks:

Although the problem appeared to be highly calculation oriented, no actual calculation was required to arrive at the first order solution. All that is needed is a step by step analysis aided by strong visualization and the realization that the geosynchronous nature of the orbit is the key input.

3.2. Problem 2 : A Tale of two Phases

3.2.1. Problem Statement:

In the year 2021 the ISRO's home-bred space shuttle identified by the code MJ-3105 is planned to dock with our space observatory 4N1K37 which orbits the earth at 600 km in circular orbit. Imagine, that due to some slight calculation errors the MJ lands in the correct orbit but at a wrong phase such that the distance between MJ and the observatory is 916 km and the observatory is ahead of MJ in its orbit. (Assume the orbit of 4N1K37 to be a non-precessing circle). As a corrective measure an appropriate elliptical orbit is suggested. If MJ-3105 is constrained by the fact that it can only apply thrust in a direction tangential to its orbit,

i. What possible orbits can it be put into such that after one complete periodof MJ in its new orbit, its location and the observatory's location will turn out to be the same? Draw a neat diagram depicting the same. (Note: After one complete revolution the corresponding thrust in the opposite direction will restore the circular orbit, thus completing this phase correction sequence.)

Figure for problem 2

ii. Find the minimum thrust required for this complete maneuver and also find the corresponding orbit.

iii. If the initial phase angle difference was 30° instead, what would be the minimum thrust?

3.2.2. Motivation:

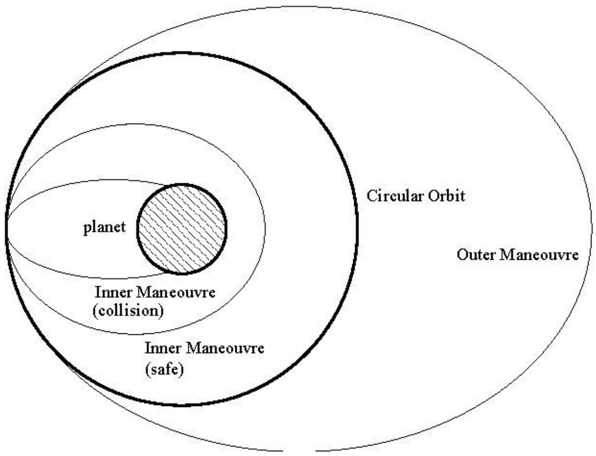
Inorder to change the phase of an object in orbit around a central body one needs to exert some sort of external force on it. This force will change the momentum of the body at that given point in the orbit. Now as a result of this new velocity the orbit itself changes defeating the whole purpose of the initial momentum. The solution to this Catch-22 situation is the selection of certain specific orbits set out in this problem.

3.2.3. Solution:

Consider the figure below. Now we know that the initial straight line distance between 4N1K37 and MJ-3105, with both in same circular orbit is 916km.

We can calculate the chord length as,

$$d = 2\sin(\theta/2) \text{ i.e. } \theta = 2sin^{-1}\left(\frac{d}{2r}\right)$$



Now let TMJ be the time taken by MJ to complete one orbit. We know by Kepler's third law that inner orbits have smaller time period and that the outer orbits will have a larger time period. Consider the orbits shown in the figure1. For 4N1K37 and MJ to come to same position, when MJ is in an outer orbit, 4N1K37 must cover an integral number of orbits less an angle θ in one orbital period of MJ. However when MJ is in an inner orbit, it must cover an integral number of orbits in time required for 4N1K37 to cover an angle 2 θ in its orbit. Hence the governing conditions for circular orbit of 4N1K37 are

Figure Different maneuvering orbits discussed in problem 2.

$$T_{MJ,outer} = \left(\frac{2(n+1)\pi - \theta}{2\pi}\right) T_{4N} n\left(T_{MJ,inner}\right) = \left(\frac{2\pi - \theta}{2\pi}\right) T_{4N}$$

By Kepler's Third law,

$$\frac{a_{MJ}^3}{T_{MJ}^2} = \frac{R_{4N}^3}{T_{4N}^2} a_{MJ} = R_{4N}\left(\frac{T_{MJ}}{T_{4N}}\right)^{\frac{2}{3}}$$

Thus we get two cases for inner and outer orbits,

$$a_{MJ,outer} = R_{4N}\left(n + 1 - \frac{\theta}{2\pi}\right)^{\frac{2}{3}} = R_{4N}\left(n + 1 - \frac{1}{\pi}sin^{-1}\frac{d}{2r}\right)^{\frac{2}{3}}$$

$$a_{MJ,inner} = R_{4N}\left(\frac{1}{n}\left(1 - \frac{\theta}{2\pi}\right)\right)^{\frac{2}{3}} R_{4N}\left(\frac{1}{n}\left(1 - \frac{1}{\pi}sin^{-1}\frac{d}{2r}\right)\right)^{\frac{2}{3}}.$$

For an elliptical orbit, using energy conservation, velocity at a distance r from the focus is given by,

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}.$$

Hence thrust per unit mass is given by,

$$P = \sqrt{GM\left(\sqrt{\frac{1}{r}} - \sqrt{\frac{2}{r} - \frac{1}{a}}\right)} P_{outer} = \sqrt{\frac{GM}{R_{4N}}}\left(\sqrt{2 - \left(n + 1 - \frac{1}{\pi}sin^{-1}\left(\frac{d}{2r}\right)\right)^{\frac{-2}{3}}} - 1\right) P_{inner}$$

$$= \sqrt{\frac{GM}{R_{4N}}}\left(1 - \sqrt{2 - \frac{1}{n}\left(1 - \frac{1}{\pi}sin^{-1}\frac{d}{2r}\right)^{\frac{-2}{3}}}\right).$$

Now note, for the outer orbits as the order of the orbit (n) increases, the thrust must increase as for the same perigee point the semi-major axis is increases. Similarly for the inner orbits as n increases for the same apogee point the semi-major axis is decreasing. Hence both the velocity difference and the thrust are more. Hence the order of both orbits should be 1. Explicitly calculating ratio of thrust to initial momentum values we see,

$$P_{outer,916km} = 0.1681 \text{ , } P_{inner,916km} = 0.0071$$

$$P_{outer,30°} = 0.1627 \text{ , } P_{inner,30°} = 0.0303$$

Thus it seems as though, both the inner orbits are favoured. As a final consistency check we calculate the ratio of the perigee distance to the radius of the earth. Note for inner orbits the current point is at apogee.

$$r_{perigee} = 2a - r_{apogee}\rho = \left(2\left(1 - \frac{1}{\pi}sin^{-1}\frac{d}{2r}\right)^{\frac{2}{3}} - 1\right)\frac{R_{4N}}{R_e}\rho_{916km} = 1.062\rho_{30°} = 0.970$$

Thus, in the second case, perigee turns out to be inside the Earth. Hence, even though the inner orbit is favoured due to low thrust, only the outer orbit can be chosen. Initial velocity is the circular velocity i.e.

$$v = \sqrt{\frac{GM}{R_{4N}}} \cdot 7.561\, km/s$$

Hence, we have $P_{inner,916km} = 53.6\, m/s$ and $P_{outer,30°} = 1230\, m/s$

3.2.4. Remarks:

The key ideas to learn from the problem are:

i. If a body applies tangential force the initial point is constrained to be either the peri-point or ap-point.

ii. In order to change the phases of the orbit we can select a few quantized orbits such that the satellite can make up for the lost phase by going around once in a new orbit.

iii. For any orbital maneuver it is imperative to check whether the modified orbit collides with the central body. This check must be externally imposed and cannot be guessed from the initial conditions.

3.3. Problem 3: The Great Escape

3.3.1. Problem Statement:

An earth-like planet 'Echor', resides in a stellar system at approximately 10 A. U. from its parent star, which is similar to the Sun. The planet has no natural satellite and other planets in that system are far from this planet. The Echorans have decided to launch their first interstellar mission to colonize other habitable planets in other star systems. The starship 'Chabya' designed for the purpose has the following dimensions. It consists of two spherical sub-ships (H and B), approximately 20km in diameter each, attached to one another by a small tether in the center. The density of the spaceships is the same as that of the planet i.e. approximately 5.5 g/cc. The first step to get the massive Chabya to the closest star is to get it out of the gravitational influence of Echor. For this purpose the ship is firstly put up in a circular orbit about Echor with radius equal to12,960,500km. The ship is oriented such that the centers of the two sub ships and the center of Echor are always collinear and Chabya-B always lies on the inside. It is believed that when an appropriate retarding impulse is provided, the spacecraft will go into a new orbit and at somepoint, Chabya-H will be able to escape outward leaving Chabya-B behind. Find the minimum retarding impulse to achieve this. (Note: The breaking strength of the tether is $7\text{x}10^{12}$ N.)

3.3.2. Motivation:

The main idea behind this problem is that for a system of rigid bodies with no relative rotation, the velocity of each body is same as that of the center of mass. However, when the individual parts separate, they continue to have the same initial velocity but may move differently based on the differential forces acting on them. In addition, for a given velocity, the orbit depends on the gravitational potential at that point. Thus, for a small change in initial position, the final positions of the bodies may be remarkably different. Another key idea involved is that parameters of the problem are such that mutual gravitational force between the bodies is not to be ignored.

3.3.3. Solution:

Firstly, consider the two satellite system. The forces acting on each component are a net inward force due to both tension and gravity. In the frame of the center of mass of the system, there is a net outward force on each body. And a differential force due to gravity. Consider the force equation for Chabya-H,

$$F_{centrifugal} = F_{gravitational} + T_{net}$$

For Chabya-B,

$$F_{centrifugal} = F_{gravitational} - T_{net}$$

Now the tension is a combination of tether tension and gravitational interaction.

$$T_{net} = T_{gravitational} + T_{tether} = \frac{Gm^2}{(2r)^2} + T_{tether} = \frac{4G\pi^2 r^4 \rho^2}{9} + T_{tether}$$

Where, r is the radius of each sub-part.
In this orbit, we have $T_{net} = 1.41 \cdot 10^{15} N$.

Now note that the mutual gravitational force between the Chabya-B and the Chabya-H in the initial orbit is much larger than the limiting breaking strength of the tether. Almost the entire contribution to the Tnet comes from the mutual gravitational force between the two components. Hence the tether cannot automatically break in this orbit. To separate the two parts automatically, one must move the satellite to a point where their mutual gravitational attraction can be overcome; or in other words move it near the Roche Limit.

Now the Roche Limit is given by,

$$d_{Roche} = R_{planet} \left(\frac{2\rho_M}{\rho_m} \right)^{\frac{1}{3}} = R_{planet} 2^{\frac{1}{3}}.$$

We give a retarding impulse to the satellite such that it goes in an elliptic orbit with the initial orbit position as the ap-point and the Roche limit distance as the peri-point. We need not go any closer, as when the tether will be at the Roche limit, Chabya-B will be inside the Roche sphere and Chabya-H will be just outside and the tension in the tether will just exceed the breaking strength. On separation of the tether, both the parts will continue to have the center of mass velocity. As such Chabya-H which is outside may overcome the gravitational potential and escape whereas Chabya-B which is inside will fall inwards.

Once again, we use the expression for velocity of any orbiting body in an elliptic orbit

$$v = \sqrt{GM \left(\frac{2}{r} - \frac{1}{a} \right)}.$$

Hence to make the Roche limit distance as the peri-point, the velocity at the ap-point should be,

$$v_{apo} = \sqrt{GM \left(\frac{2}{a(1+e)} - \frac{1}{a} \right)} = \sqrt{\frac{2GM}{R_{apo} + R_{peri}} \left(\frac{R_{peri}}{R_{apo}} \right)} v_{apo} = \sqrt{\frac{2GM}{R_{circular} + d_{Roche}} \left(\frac{d_{Roche}}{R_{circular}} \right)}$$

For the initial circular orbit,

$$v_{circular} = \sqrt{\frac{GM}{R_{circular}}}$$

Hence we have,

$$\Delta v = v_{circular} - v_{apo} = \sqrt{\frac{GM}{R_{circular}}} - \sqrt{\frac{2GM}{R_{circular} + d_{Roche}}\left(\frac{d_{Roche}}{R_{circular}}\right)}$$

$$= \sqrt{\frac{GM}{R_{circular}}}\left(1 - \sqrt{\frac{2^{\frac{4}{3}}R_{planet}}{R_{circular} + 2^{\frac{1}{3}}R_{planet}}}\right).$$

Thus the change in momentum is given by,

$$\Delta p = m_{satellite}\sqrt{\frac{GM}{R_{circular}}}\left(1 - \sqrt{\frac{2^{\frac{4}{3}}}{\frac{R_{circular}}{R_{planet}} + 2^{\frac{1}{3}}}}\right).$$

Noting that it is an 'Earth-like' planet, we use values of the Earth mass and the Earth radius. The numerical value of this change is $3.2\ 10^{19}$ kg m/s.

3.3.4. Remarks:

(i) Although the gravitational force of everyday bodies is negligible, one must consider it when body sizes are substantially large and they are placed in a weak gravitational potential.

(ii) The reason why Chabya-H leaves is not clear, immediately, however careful analysis shows us that if the split between Chabya B and H occurs at the right spot then the two will separate and the initial momentum will carry one part out but the other will fall in.

4. Summary

The above problems are meant to give students a flavor of the vast and exciting field of Orbital Dynamics. The attempt was to show that in orbital mechanics, problems which look daunting at the first-sight can be solved with nothing more than essential simple physical arguments and knowledge of pre-college mathematics. It must be noted that each of these problems stimulates the student to think in an 'out-of-the-box' manner, and emphasizes concepts rather than mechanical calculations. Exposure to such problems will hopefully attract meritorious students to the wonderful world of Astronomy.

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References

[1] Kenneth J. Adney, “Applying Kepler’s third law”, Phys.Teach., 25, 493 (1987)
[2] Manfred Bucher, “Kepler’s third law: Equal volumes in equal times”, Phys. Teach. 36, 212 (1998)
[3] Harold Cohen, “Testing Kepler’s laws of planetary motion”, Phys. Teach. 36, 40 (1998)
[4] Mario Iona, “Remember properties of conic sections”, Phys. Teach. 39, 20 (2001)
[5] Juan Lin, “A demonstration of Kepler’s third law”, Phys. Teach., 31, 122 (1993)
[6] James L. Mariner & Jack K. Horner, “Kepler’s second law and synodic period”, Phys. Teach. 19, 116 (1981)
[7] Ellis D. Noll, “Kepler’s third law for elliptical orbits”, Phys. Teach. 34, 42 (1996)
[8] A E Roy and D Clarke, “Astronomy: Principles and Practices”, 4th ed., (Institute of Physics Publishing, 2003)
[9] Michael J. Ruiz, “Kepler’s Third Law Without a Calculator”, Phys. Teach. 42, 530 (2004)
[10] Stephen B. Turcotte, “Orbital Timing for a Mission to Mars”, Phys. Teach. 43, 293 (2005)